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MANUAL
OF
SHORT METHODS IN ARITHMETIC

HUNTER

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SHORT METHODS IN ARITHMETIC

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A MANUAL

OF

SHORT METHODS IN ARITHMETIC

DESIGNED FOR THE USE OF SCHOOLS

AND TO FACILITATE THE ARITHMETICAL CALCULATIONS

OF BUSINESS AND SCIENCE

BY THE

REV. JOHN HUNTER, M.A.

10240

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PREFACE.

A VERY great amount of useful progress has been made, during the last thirty years, in the development of the power of common arithmetic to deal with many problems for which Algebra was wont, less intellectually, to be employed. Algebra has a boundless field for reasoning and operation, in which Arithmetic is only a very humble though a requisite auxiliary; but the employment of the more elementary processes of Algebra to solve arithmetical questions used to be too often resorted to, as a mechanical relief from the more useful work of purely arithmetical investigation.

Arithmetic as now taught in the best Treatises exhibits, perhaps, its highest degree of power to deal with the principles involved in the various Rules and in miscellaneous problems; but its operations still admit of considerable improvement in elegance and ease; and to promote such improvement is the aim of this Manual of Short Methods. And here let it be observed that, in many instances, the Author's *shortness* of method consists in the *easiness* of the workings rather than in the *space* they occupy.

The book is intended for the more advanced Arithmetic classes in schools, as well as for private students, and it may accompany the use of any general treatise of Arithmetic. Let general principles, by all means, be well understood before short methods of calculation are sought after; but at the same time let it be observed that there is cultivation of intellect, as well as lessening of labour, in the proper use of the ciphering artifices that are exemplified and recommended in the present publication.

It is hoped that to many of those warehouse clerks that are much occupied in extending invoices, some portion of the chapter on the Calculation of Prices will be of special service.

The aim and construction of this Manual are so peculiar that, in order to secure thoroughly the advantage to be derived from the study of it, the publication of a Key to its numerous unworked examples has been recommended. The Author has accordingly prepared one; and he hopes that the student, having first worked without its aid, will, by comparison of his own work with the method presented in the Key, have either the satisfaction of finding that his own skill is adequate, or the benefit of learning how his skill may be improved.

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MANUAL

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CHAPTER I.

THE GREATEST COMMON MEASURE.

1. The Rule for finding the Greatest Common Measure is often helpful for the reduction of a fraction's numerator and denominator to lowest terms; but it is resorted to oftener than is necessary. Fractions, seeming at first sight to require the usual process of determining the G.C.M., may in very many instances be easily reduced to lowest terms by trial, as we shall now exemplify.

Let the following be proposed for simplification :—

$$\frac{361}{627}, \quad \frac{637}{1027}, \quad \frac{234}{1001}, \quad \frac{957}{1131}, \quad \frac{10001}{11242}.$$

In the first of these we can discover no small divisor of the numerator; but if we take down the denominator by itself and break it up into factors, we shall find that it contains 3 and 209, and that 11 goes in 209 and gives 19; so we have easily ascertained that the denominator is divisible by 3, or 11, or 19. Now, as neither 3 nor 11 will go exactly in 361, we should try 19, which will be found to go 19 times; and thus the fraction is found reducible to $\frac{19}{33}$.

In the second fraction, the numerator shows itself

obviously divisible by 7, giving 91; and 91 is easily seen to be also divisible by 7, giving 13. The upper number, then, will divide by 7 or 13, but the lower number will not divide by 7; therefore, it must divide by 13, if we are to have any cancelling at all. And 13 is found to measure both terms of the fraction, reducing it to the simpler form $\frac{49}{79}$.

When we examine the third fraction, its numerator seems easier than its denominator for breaking up; we therefore take it apart and divide it successively by 2 and 3 and 3, thus obtaining 13 as a trial divisor for 1001; for neither 2 nor 3 will suit that number. Dividing the two terms by 13, we get $\frac{18}{77}$.

The fourth fraction obviously allows 3 to be struck out of both terms, reducing it to $\frac{319}{377}$; and now 11 may be seen to go in the numerator 29 times; and 29 divides both terms down to $\frac{11}{13}$.

In the last of the proposed fractions the numerator seems to resist a first trial; but the denominator plainly shows itself equal to $2 \times 5621 = 2 \times 7 \times 803 = 2 \times 7 \times 11 \times 73$; and though neither 2, nor 7, nor 11 can be struck out of the numerator, 73 being tried is found to be contained exactly 137 times; so that the resulting fraction is $\dfrac{137}{2 \times 7 \times 11} = \frac{137}{154}$.

2. The usual mode of procedure in seeking the Greatest Common Measure of two numbers might in many instances be much shortened. Compare the two subjoined ways of working *Ex.* 1.

Ex. 1. Find the G.C.M. of 5063 and 55029.

On the left we present the ordinary operation, but securing some economy of space by writing the successive remainders vertically under the first divisor, as they arise.

On the right, however, is shown a much better method. When we obtain the second remainder, 664, and have to find the G.C.M. of 664 and 4399, we can readily judge that the factor 8 may be struck out of 664, as 2 is not even once contained in 4399, and consequently

8 cannot be a factor in the required common measure. This cancelling shortens very much the rest of the work.

5063	55029	10
	50630	
4399	5063	1
	4399	
664	4399	6
	3984	
415	664	1
	415	
249	415	1
	249	
166	249	1
	166	
G.C.M. = 83	166	2
	166	

Otherwise.

5063	55029	10
	50630	
4399	5063	1*
8)664	4399	53
G.C.M. = 83	415	
	249	
	249	

Ex. 2. Find the G.C.M. of 7371 and 12441; also of 2449 and 5609.

(i.)

7371	12441	1
10)5070	7371	14
507	7098	
273	507	1
2)234	273	2
117	234	
G.C.M. = 39	117	3
	117	

(ii.)

2449	5609	2
	4898	
711	2449	3
	2133	
4)316	711	9
G.C.M. = 79	711	

Ex. 3. Find the G.C.M. of 47759 and 38957; also of 290413 and 906019.

(i.)

38957	47759	1
6)8802	38957	26
1467	2934	
	9617	
	8802	
5)815	1467	9
G.C.M. = 163	1467	

(ii.)

290413	906019	3
	871239	
20)34780	290413	
G.C.M. = 1739	1739	167
	11651	
	10434	
	12173	
	12173	

* When the quotient is 1, the divisor need not be inserted under the dividend to get the remainder.

Exercises 1.

Find the G.C.M. of—

1. 4371 and 17391.
2. 2668 and 8188.
3. 73033 and 118007.
4. 39330 and 29735.
5. 54385 and 57670.
6. 137897 and 78329.
7. 56498, 67691, and 30094.
8. 22661, 266815, and 42527.
9. 425383, 348467, and 228269.

Exercises 2.

Reduce to simplest terms the following fractions: the first eight by trial, the last four by first calculating the G.C.M. of numerator and denominator:—

1. $\frac{693}{1639}$. 2. $\frac{552}{1058}$. 3. $\frac{703}{925}$. 4. $\frac{559}{3311}$. 5. $\frac{2765}{6241}$. 6. $\frac{2538}{2773}$.
7. $\frac{3503}{4746}$. 8. $\frac{2604}{14043}$. 9. $\frac{323}{1349}$. 10. $\frac{4171}{6499}$. 11. $\frac{3287}{6631}$. 12. $\frac{2461}{13589}$.

CHAPTER II.

THE CALCULATION OF PRICES.

1. The Rule of Practice proposes to substitute shorter work for the ordinary process of compound multiplication of money, when the given multiplier is a large number. And it is very serviceable when the given multiplicand is an easy aliquot part, as 3*s.* 4*d.*, 4*s.*, 5*s.*, &c., or consists of two easy aliquots, as 8*s.* 4*d.*, 2*s.* 9*d.*, 13*s.* 4*d.*, &c.

Thus, to find the amount of 269 articles at 6*s.* 3*d.* each, the method of Practice, as shown in the annexed form of operation, cannot, perhaps, be surpassed in convenience by any other method of calculation; but if the given price of each article were 15*s.* $8\frac{1}{4}d.$, Practice supplies no simpler mode of operation than the tedious and clumsy one which we subjoin, and which,

	269 at 6*s.* 3*d.*
5*s.* 0*d.* = $\frac{1}{4}$	£67 5*s.* 0*d.*
1 3 = $\frac{1}{4}$	16 16 3
	£84 1*s.* 3*d.*

though not quite so bulky, is not so easy as the multiplying process which we place beside it for comparison :—

		269 at 15*s.* $8\frac{1}{4}$*d.*		£0 15*s.* $8\frac{1}{4}$*d.* × 5
10*s.* 0*d.*	$= \frac{1}{2}$	£134 10 0		12
5 0	$= \frac{1}{2}$	67 5 0		9 8 3
0 6	$= \frac{1}{10}$	6 14 6		11
0 2	$= \frac{1}{3}$	2 14 10		103 10 9
0 $0\frac{1}{4}$	$= \frac{1}{8}$	0 5 $7\frac{1}{4}$		2
		£210 19*s.* $11\frac{1}{4}$*d.*		207 1 6
				3 18 $5\frac{1}{4}$
				£210 19*s.* $11\frac{1}{4}$*d.*

We shall presently direct attention to a much better method than either of these, when we have first exemplified a little the mode of dealing with small given multipliers.

2. When the given number of articles of which the cost is required is 13, or under 12, and the given price is not an exact aliquot, it will generally be best to use multiplication in the ordinary way.

Ex. 1. Find the cost of 7 yards of cloth at 14*s.* 6*d.* a yard, that of 11 yards of linen at 2*s.* $5\frac{1}{4}$*d.*, and that of 13 yards of silk at 4*s.* $8\frac{1}{2}$*d.*

14*s.* 6*d.*	2*s.* $5\frac{1}{4}$*d.*	4*s.* $8\frac{1}{2}$*d.*
7	11	13
£5 1*s.* 6*d.* *Ans.*	26*s.* $9\frac{3}{4}$*d.* *Ans.*	£3 1 $2\frac{1}{2}$*d.* *Ans.*

Note.—It is to be wished that the Multiplication Table included products of 13. Let the pupil con these products, if not already familiar to him. They are as follows :—

13 times 2 are	26		13 times 8 are	104
3 „	39		9 „	117
4 „	52		10 „	130
5 „	65		11 „	143
6 „	78		12 „	156
7 „	91		13 „	169

3. When the given number of things is 12, the amount is just as many shillings as there are pence in the price.

Ex. 2. Calculate the cost of a dozen copies of a book at 4*s.* $9\frac{3}{4}$*d.* per copy, and that of a pound weight of gold at £3 17*s.* $10\frac{1}{2}$*d.* per ounce.

4*s.* $9\frac{3}{4}$*d.*	77*s.* $10\frac{1}{2}$*d.*
12	12
$57\frac{3}{4}$*s.* = £2 17*s.* 9*d.* *Ans.*	$934\frac{1}{2}$*s.* = £46 14*s.* 6*d.* *Ans.*

4. When the given number of articles is greater than 13, but less than 24, multiply by 12 + 2, 12 + 3, 12 + 4, &c. . . . 12 + 11.

Ex. 3. Find the amount of 17 things at £2 17*s.* $9\frac{1}{2}$*d.* each, and that of 23 things at 12*s.* $10\frac{3}{4}$*d.* each.

£2 17*s.* $9\frac{1}{2}$*d.* × 5	£0 12*s.* $10\frac{3}{4}$*d.* × 11
12	12
34 13 6	7 14 9
14 8 $11\frac{1}{2}$	7 1 $10\frac{1}{4}$
£49 2*s.* $5\frac{1}{2}$*d.* *Ans.*	£14 16*s.* $7\frac{1}{4}$*d.* *Ans.*

Here the advantage of first multiplying by 12 is that we say at once 12 times $9\frac{1}{2}$*d.* is $9\frac{1}{2}$ shillings, or 9*s.* 6*d.*, set down 6*d.* and carry 9*s.*; and again, 12 times $10\frac{3}{4}$*d.* is $10\frac{3}{4}$*s.*, or 10*s.* 9*d.*, set down 9*d.* and carry 10*s.*

Note.—Special instances must frequently occur, for which ways of calculating, simpler than that of a general rule, are available. If, for example, the cost of a ton at £2 13*s.* 4*d.* per cwt. is required:—Expressing the price in the single denomination of shillings, we have 20 things at $53\frac{1}{3}$ shillings each. Now, 20 at $53\frac{1}{3}$ shillings each is the same as $53\frac{1}{3}$ at 20 shillings each; that is, £$53\frac{1}{3}$, or £53 6*s.* 8*d.* is the cost required.

Exercises 3.

		£	s.	d.
1.	18 things at	0	7	$3\frac{3}{4}$
2.	19 ,, ,,	3	18	$11\frac{1}{4}$
3.	15 ,, ,,	5	19	$10\frac{1}{2}$
4.	17 ,, ,,	0	17	$8\frac{1}{4}$
5.	20 ,, ,,	0	6	5
6.	12 ,, ,,	0	13	$6\frac{1}{2}$
7.	13 ,, ,,	8	4	5
8.	21 ,, ,,	0	16	$10\frac{1}{2}$
9.	12 ,, ,,	1	10	$8\frac{1}{4}$
10.	13 ,, ,,	0	17	$9\frac{1}{2}$
11.	12 ,, ,,	0	15	$7\frac{3}{4}$
12.	21 things at	2	11	$9\frac{3}{4}$
13.	20 ,, ,,	1	10	6
14.	12 ,, ,,	9	9	10
15.	13 ,, ,,	6	13	$11\frac{1}{2}$
16.	20 ,, ,,	0	14	9
17.	15 ,, ,,	0	6	11
18.	19 ,, ,,	0	14	$5\frac{1}{4}$
19.	18 ,, ,,	3	7	$3\frac{3}{4}$
20.	22 ,, ,,	0	12	$11\frac{1}{2}$
21.	23 ,, ,,	1	16	$1\frac{1}{4}$
22.	21 ,, ,,	4	17	6

5. For those cases in which the given number of things exceeds 24, and the price is not an exact aliquot, or does not merely consist of two, or at most three, convenient aliquots, we have now an easy and concise general method to recommend; and the method will be best taught by beginning with examples in which the given multipliers are large.

Let us, then, repropose for calculation an example previously worked both by Practice and by ordinary multiplication in § 1.

Ex. 4. Calculate the cost of 269 articles at 15*s.* $8\frac{1}{4}$*d.*

The multiplier 269 is equal to $240 + 24 + 5$, hence, if we should add together 240 times, 24 times, and 5 times the given price, we should obtain the amount required.

Now, in the annexed form of calculation we use the above aggregate parts of the given multiplier in the following advantageous manner. We first obtain the product by 5 in the ordinary way; then, reducing the given price to pence, viz. $188\frac{1}{4}$ pence, we multiply that quantity by 240, by simply calling it pounds, and write 188 under the 3 pounds, and 5 shillings, for the $\frac{1}{4}$ of a pound, under the 18 shillings. We have now only to add 24 times the given price, or one-tenth of the 240 times, already obtained.

	£0	15*s.*	$8\frac{1}{4}$*d.*
			5
	3	18	$5\frac{1}{4}$
	188	5	0
$24 = \frac{1}{10}$	18	16	6
	£210	19	$11\frac{1}{4}$

Ex. 5. If a yard of linen be worth 3*s.* $7\frac{1}{4}$*d.*, what is the value of 529 yards?

Here 529 may be observed to be made up of 240, 240, 48, and 1. Reducing, therefore, the price to pence, and calling them pounds, we can calculate as follows:—

$43\frac{1}{4}$*d.* . . .	£43	5*s.*	0*d.*	=	price	× 240
	43	5	0	=	,,	× 240
$48 = \frac{1}{5} =$	8	13	0	=	,,	× 48
	0	3	$7\frac{1}{4}$	=	,,	× 1
Ans.	£95	6	$7\frac{1}{4}$	=	,,	× 529

Ex. 6. What would 11*s.* $8\frac{3}{4}$*d.* per day amount to in a year?

Here the multiplier may be divided in either of the two following ways, the latter of which will be thought by some calculators the more convenient :—

$$365 = 240 + 120 + 5;$$
$$\text{or } 365 = 240 + 80 + 30 + 15.$$

Accordingly, the calculation may be presented in the two following forms :—

	365 at 11*s.* $8\frac{3}{4}$*d.*		
			5
5	£2	18	$7\frac{3}{4}$
240	140	15	0
120 = $\frac{1}{2}$	70	7	6
365	£214	1	$1\frac{3}{4}$ *Ans.*

365	at 11*s.* $8\frac{3}{4}$*d.*		
240	£140	15	0
80 = $\frac{1}{3}$	46	18	4
30 = $\frac{1}{8}$	17	11	$10\frac{1}{2}$
15 = $\frac{1}{2}$	8	15	$11\frac{1}{4}$
365	£214	1	$1\frac{3}{4}$ *Ans.*

It should be noticed, however, that while the multiplicand here is a very awkward one for the application of the method of Practice, the multiplier 365 can be very conveniently applied in the ordinary way of compound multiplication, as being equal to $5 \times 12 \times 6 + 5$, and thus enabling us to add in the line of 5 times while multiplying by 6. We subjoin both forms of calculation.

s. *d.*	365 at 11*s.* $8\frac{3}{4}$*d.*		
10 0 = $\frac{1}{2}$	£182	10	0
1 0 = $\frac{1}{10}$	18	5	0
0 6 = $\frac{1}{2}$	9	2	6
0 2 = $\frac{1}{3}$	3	0	10
0 $0\frac{3}{4}$ = $\frac{1}{8}$	1	2	$9\frac{3}{4}$
	£214	1	$1\frac{3}{4}$ *Ans.*

£0	11	$8\frac{3}{4}$	
		5	
2	18	$7\frac{3}{4}$ =	5 times
		12	
35	3	9 =	60 ,,
		6	
£214	1	$1\frac{3}{4}$ =	(360 + 5) ,,

*** We do not see that the given price can be distributed into more convenient parts for Practice than those used above. Observe that in obtaining the last line by the second method the calculation proceeds thus :— 6 times 9 are 54 and $7\frac{3}{4}$ make $61\frac{3}{4}$ pence = 5 shillings and $1\frac{3}{4}$*d.*, set down $1\frac{3}{4}$ and carry 5, &c.

Ex. 7. Find the product of £6 11*s.* $9\frac{1}{2}$*d.* by 327.

Here 327 may be distributed into 240, 80, and 7, or

into 240, 60, 15, and 12; and hence the following choice of ways to find the required product:—

327	at £6	11*s.*	9½*d.*
	0	11	9½
			7
7	4	2	6½
240	141	10	0
80 = ⅓ =	47	3	4
£6 × 327 =	1962	0	0
	£2154	15	10½ *Ans.*

327	at £6	11	9½
	0	11	9½
240	141	10	0
60 = ¼ =	35	7	6
15 = ¼ =	8	16	10½
12 = ⅕ =	7	1	6
£327 × 6 =	1962	0	0
	£2154	15	10½

6. When the multiplier is under 240, we seek aliquots in the multiplier, always in the first instance regarding the number of pence in the price as pounds that express 240 times the price.

Ex. 8. What is the value of 217 cwt. of sugar at 35*s.* 9*d.* per cwt.?

Here we have $217 = 1 + 120 + 80 + 16$; but the price, which may at first appear to be inconvenient for Practice, has accidentally the advantage that it consists of 14*s.* and 1*s.* 9*d.*, the latter of which is one-eighth of the former, while the value at 14*s.*, or $\frac{7}{10}$ths of a pound, is easily got by multiplying £217 by 7, and doubling the first figure that arises for shillings.

We will, therefore, show the two methods, allowing that this is a question for which the method of Practice is rather better than the other way.

	217 at 35*s.* 9*d.*		
240	£429		
1	1	15	9
120 = ½ =	214	10	0
80 = ⅓ =	143	0	0
16 = ⅕ =	28	12	0
217	£387	17*s.*	9*d.* *Ans.*

	217 at £1	15*s.*	9*d.*
	217	0	0 × ·7
14*s.* 0*d.* = $\frac{7}{10}$ =	151	18	0
1 9 = ⅛ =	18	19	9
	£387	17*s.*	9*d.*

7. When the multiplier is seen to want only 1 of being a number convenient for distribution, we may

assume the convenient number and from its resulting amount deduct the given price.

Ex. 9. What sum will buy 159 articles at the average price of 7*s.* $11\frac{1}{4}$*d.* each?

Here we have $159 = 120 + 40 - 1$; but it is also equal to $120 + 24 + 15$. We will exhibit each of the processes thus suggested.

159 at £0	7*s.*	$11\frac{1}{4}$*d.*	
240	£95	5	0
$120 = \frac{1}{2} =$	47	12	6
$40 = \frac{1}{3} =$	15	17	6
160	63	10	0
159	£63	2*s.*	$0\frac{3}{4}$*d.* *Ans.*

159 at £0	7*s.*	$11\frac{1}{4}$*d.*	
240	£95	5	0
$120 = \frac{1}{2} =$	47	12	6
$24 = \frac{1}{5} =$	9	10	6
$15 = \frac{1}{8} =$	5	19	$0\frac{3}{4}$
159	£63	2*s.*	$0\frac{3}{4}$*d.*

Ex. 10. A person bought 100 articles at 13*s.* $7\frac{1}{2}$*d.* each, and sold them at 15*s.* $4\frac{1}{2}$*d.* each; what did he give for 87 of them, and what did he get for 74 of them?

For this question we have $87 = 80 + 8 - 1$, and $74 = 60 + 15 - 1$; and hereby the following modes of working are suggested.

87 at £0	13*s.*	$7\frac{1}{2}$*d.*	
240	£163	10	0
$80 = \frac{1}{3} =$	54	10	0
$8 = \frac{1}{10} =$	5	9	0
88	59	19	0
87	£59	5*s.*	$4\frac{1}{2}$*d.* *Ans.*

74 at £0	15*s.*	$4\frac{1}{2}$*d.*	
240	£184	10	0
$60 = \frac{1}{4} =$	46	2	6
$15 = \frac{1}{4} =$	11	10	$7\frac{1}{2}$
75	57	13	$1\frac{1}{2}$
74	£56	17*s.*	9*d.* *Ans.*

Exercises 4.

		£	*s.*	*d.*
1. 260	at	0	15	2
2. 264	,,	0	9	$1\frac{1}{2}$
3. 320	,,	0	4	$11\frac{1}{4}$
4. 247	,,	0	13	7
5. 344	,,	0	16	$3\frac{1}{4}$
6. 340	,,	1	8	5
7. 136	,,	1	9	8

		£	*s.*	*d.*
8. 368	at	0	10	$8\frac{1}{2}$
9. 197	,,	2	14	10
10. 294	,,	0	17	8
11. 271	,,	0	5	$3\frac{1}{4}$
12. 119	,,	0	13	11
13. 287	,,	0	18	10
14. 93	,,	3	17	$10\frac{1}{2}$

		£	s.	d.			£	s.	d.
15. 291	at	1	9	$7\frac{1}{2}$	28. 365	at	0	4	$9\frac{1}{2}$
16. 366	,,	5	13	0	29. 855	,,	4	6	$4\frac{1}{2}$
17. 184	,,	0	19	5	30. 299	,,	0	11	$9\frac{3}{4}$
18. 79	,,	0	4	$9\frac{3}{4}$	31. 547	,,	0	12	$11\frac{3}{4}$
19. 89	,,	0	5	$1\frac{3}{4}$	32. 351	,,	4	17	7
20. 375	,,	0	12	$7\frac{3}{4}$	33. 390	,,	1	16	$10\frac{1}{4}$
21. 372	,,	0	7	9	34. 558	,,	3	5	8
22. 127	,,	3	13	2	35. 503	,,	1	1	$8\frac{1}{2}$
23. 348	,,	0	6	$9\frac{3}{4}$	36. 615	,,	2	13	6
24. 411	,,	3	6	5	37. 365	,,	0	15	$8\frac{1}{4}$
25. 265	,,	0	11	$2\frac{1}{4}$	38. 139	,,	1	16	$2\frac{1}{2}$
26. 113	,,	1	12	7	39. 857	,,	2	11	4
27. 452	,,	0	19	$3\frac{1}{2}$	40. 169	,,	1	14	3

8. We have now to direct attention to some expedient methods of dealing with certain small prices of frequent occurrence in business.

In many commercial houses there is daily demand for the extension of invoices requiring such calculation as that of

135 (yds., lbs., &c.) at $22\frac{5}{8}d.$,
or $86\frac{3}{8}$,, ,, ,, ,, $2s.\ 5\frac{1}{2}d.$,
or $216\frac{3}{4}$,, ,, ,, ,, $9\frac{7}{8}d.$

In general, we should expect the best method of doing such work to be suggested by the form of the particular price or quantity; and, therefore, we shall here supply a variety of worked examples, sufficient to beget an aptitude for easy and expert calculation with such quantities and prices generally.

Twelfths of a penny sometimes occur among the prices customary in many commercial houses, but not so often as eighths. Let the learner, therefore, acquire readiness in expressing eighths of a pound as amounting to so many half-crowns, and twelfths of a pound as so many times 1*s.* 8*d.*

Ex. 11. Calculate, to the nearest farthing, 135 yards, at $22\frac{5}{8}d.$ per yard.

Here there can be no better way of working than calling the pence pounds, and dividing 135 into 120 and 15.

We set down the work as an extending clerk might do it on his rough-work paper, dividing by 2 for the 120, and by 8 for the 15, as taught in former examples. The pence arising in the division by 8 is $3\frac{3}{8}d.$, and the nearest farthing is, in favour of the buyer, $\frac{2}{8} = \frac{1}{4}d.$; in favour of the seller it would be $\frac{4}{8} = \frac{1}{2}d.$

	£22	12s.	6d.
120	11	6	3
15	1	8	$3\frac{1}{4}$
Ans.	£12	14s.	$6\frac{1}{4}d.$

Ex. 12. Find, to the nearest farthing, the cost of 86 lb. 6 oz. of tea, at 2s. $5\frac{1}{2}d.$ per lb.

$$\begin{array}{rrrr} \frac{1}{8} \text{ of } £86\ 7s.\ 6d. = & £10 & 15s. & 11\frac{1}{4}d. \\ 43\frac{1}{4}d. = & 0 & 3 & 7\frac{1}{4}d. \\ \hline \textit{Ans.} & £10 & 12s. & 4d. \end{array}$$

Here, as the price wants only a halfpenny of half-a-crown, we first assume 2s. 6d. for the price, making the amount equal to an eighth of $£86\frac{3}{8}$; then we have to deduct for $86\frac{3}{8}$ lb. at a halfpenny, which comes to $43\frac{3}{16}$ pence, and the nearest farthing to $\frac{3}{16}d.$ is $\frac{4}{16} = \frac{1}{4}d.$; so we subtract $43\frac{1}{4}d.$

Ex. 13. Calculate, to the nearest farthing, $216\frac{3}{4}$ yards, at $9\frac{7}{8}d.$ per yard.

$$\begin{array}{lll} 216\frac{3}{4} \text{ at } 10d. = 2167\frac{1}{2}d. & = 180s. & 7\frac{1}{2}d. \\ \frac{1}{8} \text{ of } 216\frac{3}{4}d. = 27d. & = \quad 2 & 3 \\ \cline{2-3} \textit{Ans.}\ £8\ 18s.\ 4\frac{1}{2}d. & \quad 178 & 4\frac{1}{2} \end{array}$$

Here again the calculation is accidentally made easy, as $9\frac{7}{8}$ is only $\frac{1}{8}$ short of 10. The eighth of $216\frac{3}{4}d.$ is $27\frac{3}{32}d.$, where the value of the fraction is much below one farthing, and therefore is neglected.

Ex. 14. What will 377 lb. of cotton come to at $10\frac{1}{8}d.$ per lb.?

$$\begin{array}{l} 3770d. \\ \quad 47\frac{1}{8} \\ \hline 3817d. = 318s.\ 1d. = £15\ 18s.\ 1d.\ \textit{Ans.} \end{array}$$

Here 377 is not made up of a convenient set of aggregate parts: calculation by means of 240 + 48 + 48 + 40 + 1, though it would be easy, would take up too much time. We have simply multiplied 377d. by

$(10 + \frac{1}{8})$, and reduced to shillings and pounds successively in a continuous line, not showing the divisors 12 and 20.

Ex. 15. Calculate $38\frac{1}{4}$ yards at $17\frac{5}{12}d.$ per yard.

	£	s.	d.
	£17	8*s.*	4*d.*
24	1	14	10
12	0	17	5
2	0	2	$10\frac{3}{4}$
$0\frac{1}{4}$	0	0	$4\frac{1}{4}$
	£2	15*s.*	6*d.* *Ans.*

(*Otherwise.*)

	£	s.	d.
	£0	3*s.*	$2\frac{1}{4}d.$ × (12 + 5)
	1	18	3
12)	0	15	$11\frac{1}{4}$
	0	1	$3\frac{3}{4}$
	£2	15*s.*	6*d.* *Ans.*

By the first of these methods we begin with $£17\frac{5}{12}$ as the value of 240 yards; then we take 24 yards $= \frac{1}{10}$, 12 yards $= \frac{1}{2}$, 2 yards $= \frac{1}{6}$, and $\frac{1}{4}$ yard $= \frac{1}{8}$.

By the second method we regard the question as requiring the cost of $17\frac{5}{12}$ yards at $38\frac{1}{4}d.$ or 3*s.* $2\frac{1}{4}d.$ per yard, and first taking 12 times the price $= 38\frac{1}{4}$ shillings = £1 18*s.* 3*d.*, we add 5 times 3*s.* $2\frac{1}{4}d.$ as the cost of 5 yards, and $\frac{1}{12}$ of that as the cost of $\frac{5}{12}$ of a yard.

There is little, if any, superiority in either of the methods over the other.

Ex. 16. Find the cost of $185\frac{3}{4}$ yards at $20\frac{1}{2}d.$ per dozen yards, and also at $27\frac{1}{4}d.$ per dozen yards.

$1857\frac{1}{2}d.$
$1857\frac{1}{2}$
$92\frac{7}{8}$
12)3808*d.*
$317\frac{1}{4}d.$
= 26*s.* $5\frac{1}{4}d.$ *Ans.*

	£	s.	d.
	£2	5*s.*	5*d.*
120	1	2	$8\frac{1}{2}$
60	0	11	$4\frac{1}{4}$
6	0	1	$1\frac{1}{2}$
	1	15	$2\frac{1}{4} - \frac{1}{2}d.$
	£1	15*s.*	$1\frac{3}{4}d.$ *Ans.*

For the first part of the above question we have multiplied 185·75*d.* by $(10+10+\frac{1}{2})$, and then divided by 12 for dozens. This reserving of the 12 till the last should be a general practice when neither the price nor the number of articles is convenient for division by 12 at the outset.

But for the second part of the question, which we have worked by the method of calling the pence pounds, we can conveniently begin with a twelfth part of £27 5*s.*,

viz. £2 5*s*. 5*d*. Then assuming 186 for the number of yards $= 120+60+6$, or $\frac{1}{2}$, $\frac{1}{2}$ and $\frac{1}{10}$, we have only to deduct the value of $\frac{1}{4}$ of $\frac{1}{12}$ of $27\frac{1}{4}d.$, which for the nearest approximation is $\frac{1}{48}$ of $24d. = \frac{1}{2}d.$

*** As examples like those with which we are now dealing cannot be referred to any general Rule, we must trust to the exhibition of a variety of specimen workings appropriate to particular questions as the best means of promoting general skill in the learner. We will therefore extend preparatory illustration a little farther, before prescribing the next set of Exercises.

Ex. 17. What will $35\frac{3}{4}$ yards of net come to at $16\frac{8}{12}d.$ per yard?

The simplest way of calculation here is that which regards $35\frac{3}{4}$ yards at $16\frac{8}{12}d.$ as equivalent to $16\frac{8}{12}$ yards at $35\frac{3}{4}d.$ We will work out this together with another method, which, though not so convenient in the present instance, would in many cases be more suitable; it takes 4 times the yards at a fourth of the price, and calls the pence pounds.

	2*s*.	$11\frac{3}{4}d.$	
		8	
1	3	10	
12)1	3	10	
	1	$11\frac{5}{6}$	
£2	9*s*.	8*d*.	*Ans*.

143 at $4\frac{1}{6}d.$				
	£4	3*s*.	4*d*.	
120	2	1	8	
20	0	6	$11\frac{1}{3}$	
3	0	1	$0\frac{1}{2}$	
	£2	9*s*.	8*d*.	*Ans*.

Ex. 18. Find the value of $25\frac{1}{2}$ yards at 3*s*. $10\frac{5}{12}d.$ per yard.

There can be no better way for this question than to multiply the given price by $(1+12+12+\frac{1}{2})$ which are aggregate parts of $25\frac{1}{2}$. The convenience of the multiplier 12, converting, as it does, the pence into $10\frac{5}{12}$ shillings, is obvious.

£0	3*s*.	$10\frac{5}{12}d.$	
2	6	5	
2	6	5	
0	1	$11\frac{5}{24}$	
£4	18*s*.	$7\frac{1}{2}d.$	*Ans*.

Ex. 19. What is the value of a mixture of $13\frac{3}{8}$ gallons of spirits at 17*s*. 9*d*. and $8\frac{5}{8}$ gallons at 19*s*. 7*d*.? and what is the mixture worth per gallon?

$17s.\ 9d. \times (10 + 3 + \frac{3}{8})$	$19s.\ 7d. \times (3 + 5 + \frac{5}{8})$
8 17 6	2 18 9
8)2 13 3	8)4 17 11
0 6 $7\frac{7}{8}$	0 12 $2\frac{7}{8}$
£11 17s. $4\frac{7}{8}d.$	£8 8s. $10\frac{7}{8}d.$

Hence, value of 22 gallons = £20 6s. $3\frac{3}{4}d.$ *Ans.*

„ „ 11 „ = 10 3 $1\frac{7}{8}$

„ „ 1 „ = 18s. $5\frac{5}{8}$ *Ans.*

Exercises 5.

Calculate to the nearest farthing—

Yards		d.	Yards		d.
1. 210	at	$14\frac{7}{8}$	12. 28	at	$8\frac{3}{8}$
2. $115\frac{1}{2}$	„	$9\frac{7}{12}$	13. 52	„	$21\frac{11}{12}$
3. 317	„	$6\frac{3}{8}$	14. $19\frac{1}{4}$	„	$14\frac{5}{8}$
4. $133\frac{1}{2}$	„	$7\frac{1}{4}$	15. $43\frac{3}{4}$	„	$23\frac{7}{8}$
5. $131\frac{1}{4}$	„	$9\frac{1}{12}$	16. $37\frac{1}{2}$	„	$31\frac{7}{12}$
6. 358	„	$5\frac{3}{4}$	17. $29\frac{3}{4}$	„	$21\frac{1}{4}$
7. 280	„	$7\frac{1}{8}$	18. $37\frac{5}{8}$	„	$13\frac{3}{8}$
8. 171	„	$13\frac{3}{4}$	19. $19\frac{1}{4}$	„	$9\frac{1}{8}$
9. $276\frac{1}{4}$	„	$5\frac{5}{8}$	20. $90\frac{7}{8}$	„	$8\frac{3}{4}$
10. $156\frac{5}{8}$	„	$2\frac{3}{4}$	21. $31\frac{1}{2}$	„	$23\frac{5}{8}$
11. $113\frac{3}{4}$	„	$16\frac{5}{12}$	22. $47\frac{1}{2}$	„	$27\frac{1}{12}$

23. Find the value of $190\frac{7}{8}$ yards of lace at 1s. $11\frac{1}{2}d.$ per dozen yards.

24. What is the cost of $89\frac{1}{4}$ yards at 2s. $5\frac{1}{8}d.$ per dozen yards?

CHAPTER III.

SHORT METHODS IN SIMPLE MULTIPLICATION.

1. *To multiply a number by* 25.—This multiplier is one-fourth of 100; therefore the amount of 25 times a number is one-fourth of 100 times the number.

Hence, if you conceive two ciphers annexed to the number, and divide by 4, you will obtain 25 times the number.

Ex. 1. Multiply 298, 437, and 147, severally, by 25.

	4)298 . .	4)437 . .	4)147 . .
Answers,	7450,	10925,	3675.

Note.—Similarly, to multiply by 125, divide 1000 times by 8.

2. *To multiply a number by* 75.—This multiplier is three-fourths of 100, or one-fourth less than 100; hence the amount of 75 times a number must be one-fourth less than 100 times the number.

Conceive, therefore, two ciphers annexed to the number, divide by 4, and subtract, and you will obtain 75 times the number.

Ex. 2. Multiply 365, 5278, and 1527, severally, by 75.

	4)365. .	4)5278. .	4)1527. .
	9125	131950	38175
Answers,	27375,	395850,	114525.

3. When a multiplier begins or ends with 25, the short process described in § 1 may be used.

Ex. 3. Multiply 1737, severally, by 825 and 258.

1737	1737
825	258
43425	43425
13896	13896
1433025 *Ans.*	448146 *Ans.*

4. *To multiply a number by* 99, 999, *&c.*—This is to take once less than 100, 1000, &c. times the number.

Hence, by annexing two, three, &c. ciphers to the given multiplicand, and then subtracting the given multiplicand, you will obtain the required product.

Ex. 5. Multiply 467, severally, by 99 and 999.

46700	467000
467	467
46233 *Ans.*	466533 *Ans.*

Note.—By reference to this kind of abridgment a

common rule given in treatises on Arithmetic, about recurring decimals, may be illustrated.

Suppose it is required to convert $\cdot 27\dot{6}829\dot{2}$ to a vulgar fraction.

The usual Rule for such conversion is as follows :—

To find the numerator : From the given decimal considered as an integer subtract the non-repeating part considered as a separate integer. Then to form the denominator : Write as many nines as the circulating period contains figures, and annex as many ciphers as the non-repeating part contains figures.

Following this Rule we have

$$\frac{2768292-27}{9999900}=\frac{307588-3}{1111100}=\frac{61517}{222220};$$

then dividing the terms by their greatest common measure, 271, we obtain $\frac{227}{820}$ as the required fraction in its simplest form.

The reason of the Rule will be seen from the following demonstrative illustration :—

$$\cdot 27\dot{6}829\dot{2}=\frac{1}{100}\text{ of } 27\,\frac{68292}{99999}$$

$$=\frac{1}{9999900}\text{ of } [27\times(100000-1)+68292]$$

$$=\frac{1}{9999900}\text{ of } (2700000+68292-27).$$

And we shall generally do better than what the Rule dictates, if we adopt the first line of the above demonstrative process, and try to simplify the vulgar fraction that first arises ; that fraction in the present instance is

$$\frac{68292}{99999}=\frac{7588}{11111}=\frac{28}{41};$$

hence $\frac{1}{100}$ of $27\frac{28}{41}=\frac{1135}{4100}=\frac{227}{820}$, as before.

5. It very often happens, when a multiplier is a large number, that one of the partial products enables us to obtain easily in one line the sum of two other partial products. In the examples which we are about to give of this source of abridgment, we of course have to frame

multipliers to show the facilities; but experience proves accident to be, in at least seven cases out of ten, favourable to the application of one or another of the shortening devices which we are about to exemplify; so that these devices are well worth noting.

(*a*) When in the multiplier there occur two contiguous figures that could not easily be used at once, but that express an exact multiple of another figure, higher than unity, already used, the product by the figure already used, multiplied by the other factor of the multiple, will obtain the work of the two figures in one line.

Ex. 6. Multiply 39478 by 486, and 37409 by 763.

```
  39478              37409
    486                763
 ------             ------
 236868 × 8         261863   × 9
1894944             2356767
--------            --------
19186308 Ans.       28543067 Ans.
```

Here the figures 48 in the first multiplier express 8 times the 6; having therefore worked the line of 6 times, we take 8 times that line for 48 times the given multiplicand, observing to place the first figure, 4, under the units of its understood multiplier, 48.

In the second multiplier, the figures 63 express 9 times the 7; having therefore first worked the line of 7 times, we take 9 times that line for 63 times the given multiplicand, observing to place the first figure, 7, under the units of its understood multiplier, 63.

Ex. 7. Find the product of 9865 by 728.

```
   9865              9865
    728               728
  -----             -----
  78920 × 9         69055   × 4
 710280              276220
 -------            -------
 7181720 Ans.       7181720 Ans.
```

Here we have the choice of beginning to multiply from the right of the multiplier or from the left; for 72 is 9 times 8, and 28 is 4 times 7.

Ex. 8. Multiply 597867 by 8472.

```
  597867                  597867
    8472                    8472
 -------                 -------
 1195734                4782936     × 9
  4185069   × 12         2391468
50220828                   43046424
----------              ----------
5065129224 Ans.         5065129224 Ans.
```

Here, again, we have the choice of beginning to multiply from either end, since 12 times 7 is 84 and 9 times 8 is 72.

(*b*) Sometimes it is expedient to make the proposed multiplicand and multiplier exchange places.

Ex. 9. Multiply 19036 by 8795, and 35756 by 37908.

```
    8795                 37908
   19036                 35756
 -------               -------
 8795                  265356
 79155     × 4           2122848 = 7 times × 8
   316620              1326780   = 7   ,,   × 5
---------              ----------
167421620 Ans.         1355438448 Ans.
```

Here the proposed multiplier does not present any advantage in either case; but by transposing the factors we can shorten the work in both cases.

(*c*) Sometimes conceiving a figure of the multiplier to be lessened, and the portion taken away to be annexed to the next figure in the multiplier secures advantage.

Ex. 10. Multiply 8367 by 492, and 87058 by 7854.

Here the first multiplier $= 480 + 12$, and the second $= 6000 + 1800 + 54$; accordingly, in the former case we may take 12 times and 12 times $\times$ 4, and in the latter case we may take 6 times, 6 times $\times$ 3, and 18 times $\times$ 3.

```
    8367                  87058
     492                   7854
 -------               --------
  100404 = 12 times    522348     =  6 times
 401616  = 48   ,,     1567044    = 18   ,,
 -------                 4701132  = 54   ,,
4116564 Ans.           ---------
                       683753532 Ans.
```

Ex. 11. Multiply 175096 by 9724.

Here we have a choice of two equally good artifices.

175096		
9724		
1575864	= 9	times
12606912	= 72	„
700384	= 4	„
1702633504 *Ans.*		

175096		
9724		
700384	= 4	times
2101152	= 12	„
16809216	= 96	„
1702633504 *Ans.*		

Ex. 12. Multiply 36981 by 4697.

Here, again, we have a choice of methods; we may resolve 4697 into 4200 + 490 + 7, or we may transpose the factors.

36981		
4697		
258867	= 7	times
1812069	= 49	„
1553202	= 42	„
173699757 *Ans.*		

4697		
36981		
42273	= 9	times
380457	= 81	„
169092	= 36	„
173699757 *Ans.*		

Ex. 13. Find the product of 72996 and 6579.

Here we may make either of the numbers multiplier with advantage; for we have 72996=70000+2800+196, the last of these being 7 times 28; and we have 6579 = 6300 + 270 + 9.

72996		
6579		
656964	= 9	times
1970892	= 27	„
4598748	= 63	„
480240684 *Ans.*		

6579		
72996		
46053	= 7	times
184212	= 28	„
1289484	= 196	„
480240684 *Ans.*		

Ex. 14. Multiply together the treble of 96012, the double of 52869, and the half of 365.

This, in effect, requires to multiply the continual product of the three numbers by $3 \times 2 \times \frac{1}{2}$, that is, by 3. We shall take 365 as equal to 10 + 5 + 350.

```
  52869
  96012
 634428 × 8
5075424
-----------
5076058428
```

```
2)50760584280
   25380292140 × 7
177662044980
-------------------
1852761326220 × 3
-------------------
5558283978660 Ans.
```

(*d*) Various other devices might be exemplified, in the contrivance of which practice and experience promote expertness and save labour. We will work only one example more.

Ex. 15. Multiply 4 times 5869 by 227825.

It is often useful to remember that numbers ending with 25 or 75, when multiplied by 4, give a product ending with two ciphers. As, therefore, the product required in the present question is equal to $227825 \times 4 \times 5869 = 911300 \times 5869$, the work is thus found to be more simple than by following the direct order of the question. Moreover, we have here an accidental advantage for those who can readily multiply at once by 13, for 91 is 7 times 13. The operation will then be as follows :—

```
                   5869
227825 × 4 =     911300
               ---------
                7629700 × 7
               53407900
               ----------
               5348419700 Ans.
```

Exercises 6.

	Multiply				Multiply		
1.	62057	by	248	14.	8297 × 365	by	321
2.	do.	,,	549	15.	511736	,,	36120
3.	807439	,,	426.	16.	993865	,,	51236
4.	do.	,,	819	17.	83754	,,	2643
5.	148703	,,	726	18.	9738 × 8379	,,	25
6.	do.	,,	497	19.	9586 × 6954	,,	99
7.	27738	,,	742	20.	56749	,,	8759
8.	do.	,,	856	21.	12786 × 2786	,,	125
9.	2507386	,,	654	22.	630728	,,	187548
10.	7363891	,,	648	23.	42308 × 7854	,,	5296
11.	891257	,,	3204	24.	13296	,,	54387
12.	3926 × 8723	,,	25	25.	4962 × 9356	,,	365
13.	9819 × 7465	,,	99	26.	170625 × 9587	,,	8

CHAPTER IV.

SHORT METHODS IN DIVISION.

1. It is familiarly known that when a dividend and its divisor contain a common factor, cancelling or striking out that factor from each, before dividing one by the other, will cause no alteration of value in the result.

But, except in cases where cancelling can be used, the application of short methods to division is very limited. In the abridgments that we shall teach in the present chapter we shall deal only with dividends that contain remainders; though of course the methods of abridgment will be applicable generally. We shall divide remainders decimally.

2. *To divide a number or a simple quantity by* 25, *or* 125, *or* 75.—Multiply by the reciprocal of the divisor, viz. by ·04, ·008, or ·01$\frac{1}{3}$.

Ex. 1. Divide 51067, severally, by 25, 125, and 75.

51067	51067	51067 × ·01$\frac{1}{3}$
·04	008	17022$\frac{1}{3}$
2042·68 *Ans.*	408·536 *Ans.*	680·89$\frac{1}{3}$ *Ans.*

3. *To divide a number by* 99.—Add the quotients obtained by dividing by 100 as often, in succession, as will ensure the accuracy of the second decimal figure (if any) in the result.

Note.—For the division of a simple quantity containing a decimal fraction, the process must be carried so far as to ensure the accuracy of the second decimal place beyond the number of places in the given decimal.

Ex. 2. Divide 532701623 and 762·218, severally, by 99.

5327016·23	7·62218
53270·162	·076222
532·702	·000762
5·327	·000007
·053	7·69917
5380824·47	

The Answers therefore are $5380824{\cdot}4\dot{7}$ and $7{\cdot}6991\dot{7}$, for 47 and 17 would be found to be the remainders, if we divided by 99 in the ordinary way, not carrying the division beyond the figures given.

4. The number 365, being the number of days in a year, occurs frequently, in simple interest and other calculations, as a divisor; and the following easy Rule has been investigated to lessen the labour of the division.

To divide a number or a simple quantity by 365.—Multiply by ·002, and to the product add $\frac{1}{3} + \frac{1}{30} + \frac{1}{300}$ of itself; the sum diminished by its 10000th part will give an extremely near approximation to perfect accuracy.

Ex. 3. Divide 27493 by 365.

Ex. 4. If a person's income is £248 16*s.* 6*d.* a year, how much is it per day?

	(i.)		(ii.)
	27493		4976·5*s.*
	·002		·002
3	54·986	3	9·9530
10	$18{\cdot}32866\frac{2}{3}$	10	$3{\cdot}31766\frac{2}{3}$
10	$1{\cdot}83286\frac{2}{3}$	10	$\cdot33176\frac{2}{3}$
	$\cdot18328\frac{2}{3}$		$\cdot03317\frac{2}{3}$
10000	75·330820	10000	13·63561
	·007533		·00136
	75·323287 *Ans.*		13·63425*s.*
			= 13*s.* 7·61*d.* *Ans.*

Note.—It should be noticed that the above operations, though they make a bulky appearance, are extremely easy, being very little more than a succession of mere copyings; and for practical purposes the decimals need not be extended to the degree of accuracy to which we have carried them. For such questions as the 4th, which requires to divide a given sum of pounds, shillings, and pence by 365, an answer true to the nearest farthing will always represent the daily income as accurately as our money denominations will permit, and hence the given pence may be neglected.

In working the 4th example, then, the sum to be divided by 365 is only 4976 shillings, and the operation should be conducted in the approximative way here shown, dispensing with the deduction of the 10000*th*, and finding the answer to be simply 13*s.* $7\frac{1}{2}d.$ per day.

	4976*s.* × ·002
3	9·952
10	3·317
10	·332
	·033
	13·634*s.* = *Ans.*

As this method of finding daily from yearly income will often be found very useful in practice, we will give two more worked examples.

Ex. 5. A man's yearly income is £1359 15*s.*; how much is that per day?

Ex. 6. Find the average daily rate of an income amounting to £939 9*s.* 10*d.* in a year.

(i.)

	27195*s.* × ·002
3	54·390
10	18·130
10	1·813
	·181
	74·514*s.*
	12
	6·068*d.*
	or, £3 14*s.* 6*d.* *Ans.*

(ii.)

	18789*s.* × ·002
3	37·578
10	12·526
10	1·253
	·125
	51·482*s.*
	12
	5·784*d.*
	or, £2 11*s.* $5\frac{3}{4}d.$ *Ans.*

In the investigation of the Rule we used the reciprocal of 365, for, *dividing* by any number will produce the same result as *multiplying* by the reciprocal of the number.

Now, the reciprocal of 365, viz., $\frac{1}{365}$, is found to be equal to $\cdot00274 - \frac{10}{365}$ of $\frac{1}{100000} = \cdot00274 - \frac{1}{10000}$ of $\frac{1}{365} = \cdot00274 - \frac{1}{10000}$ of $\cdot00274$ very nearly.

Again, $\cdot00274$ is equal to $\cdot002 + \cdot00066\frac{2}{3} + \cdot00006\frac{2}{3} + \cdot00000\frac{2}{3}$, that is, equal to $\cdot002\ (1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300})$.

Hence the Rule.

5. It is often required to convert a large number of cubic inches to gallons, the divisor for which is 277·274, the number of cubic inches in a gallon; and we have investigated a Rule similar to the preceding, which makes

division by 277·274 very nearly as easy. The Rule is as follows :—

To divide a number or a simple quantity by 277·274. —Multiply by ·003, and to the product add $\frac{1}{5} + \frac{1}{500} + \frac{1}{5000} - \frac{1}{50000}$ of itself.

The reciprocal of 277·274, as worked out by ordinary division, is ·00360654, and the Rule now given will be found to be verified by working out the reciprocal according to its dictates, as required in the first of the two following questions.

Ex. 6. Find by the above Rule the reciprocal of 277·274.
Ex. 7. In 4873956 cubic inches of water how many gallons?

	(i.)
5	·00300000
100	·00060000
10	·00000600
− 10	·00000060
	·00360660
	− ·00000006
	·00360654 *Ans.*

	(ii.)
	4873956 c. in.
	·003
5	14621·868
100	2924·373
10	29·244
− 10	2·924
	17578·409
	− ·292
Ans.	17578·117 gall.

Exercises 7.

1. What weight of salmon at 2*s.* 1*d.* per lb. is worth £7 16*s.* 6*d.*?

2. On what quantity of brandy would the duty amount to 6 guineas, at the rate of 10*s.* 5*d.* a gallon?

3. If 693$\frac{2}{5}$ cubic feet of clay can be dug out in 25 hours, how much can be dug out in 24 hours?

4. The produce of a field of 75 acres is 256$\frac{5}{8}$ quarters of wheat; how many bushels is the produce per acre?

5. Calculate the 75th part of 25·934, and the 99th part of 8550828.

6. Calculate the 365th part of 25908, and the 99th part of 123456789.

7. What daily rate of expenditure amounts to £120 6*s.* 8*d.* in a year?

8. In 76880·9 cubic inches of oil how many gallons?

9. Divide 117·918 ounces by $8\frac{1}{3}$ ounces, and by $62\frac{1}{2}$, severally.

10. If a man's yearly income amount to £704 10*s.* 6*d.*, what is his daily rate of income?

11. What decimal quantity multiplied by $9\frac{3}{8}$ will produce ·01234?

12. Divide 1234 by 365, and the result by ·75.

13. How much per day is a yearly income of £350 11*s.*?

14. A cask contains 9999 cubic inches of liquid: how many gallons does it contain?

15. Calculate the 99th part of 6477778.

16. Divide 111111 farthings by 7*s.* $7\frac{1}{4}$*d.*

17. A cistern holds 5000 cubic inches of water: reduce its contents to gallons.

18. A clerk's salary is £35 10*s.* a quarter: how much is it per day?

19. What number multiplied by ·0365 will produce 30391?

20. Divide 2681217 by 99, and the result by 365.

21. A rectangular tank, 19·58 feet long and 14·07 feet wide, contains $5\frac{5}{8}$ feet depth of water. Find the volume of water in gallons.

22. Find the 99th part of the square of 3·65.

CHAPTER V.

SHORT METHODS IN THE CALCULATION OF SIMPLE INTEREST FOR DAYS.

1. In the calculation of simple interest for days, the number 365 occurs as a divisor, and we may therefore in such calculation avail ourselves of the short method of dividing by 365 exemplified and explained in the preceding chapter. Also, as a given number of days occurs as a multiplier, its employment can often be simplified by the use of easy divisors as taught in the first chapter.

And it should be noticed that in using such divisors we can limit approximation with very few figures, because we work, not from right to left, under the necessity of writing several useless figures, but from left to right, with assurance of the accuracy of each figure to wherever we extend the approximation.

Ex. 1. Calculate the simple interest on £193 15*s.* 7*d.*, for 313 days, at $4\frac{1}{4}$ per cent. per annum.

	3875*s.* 7*d.*
	£465·07 × $4\frac{1}{4}$
	1860·28
	116·2675
240	1976·5475
60	494·1369
12	98·8274
1	8·2356
313	2577·7474 ÷ 365

	£2577·747 × ·002
3	5·15549
10	1·71850
10	·17185
	·01718
10000	7·06302
	·00070
	£7·06232
	or £7 1*s.* 2·957*d.* *Ans.*

Note.—At the outset we have called the pence pounds, and divided by 100 by inserting the decimal point.

Ex. 2. Find the simple interest on £741 4*s.* 10*d.* for 1 year 203 days, at $3\frac{3}{4}$ per cent. per annum.

	14824*s.* 10*d.*
	£1778·98 × $3\frac{3}{4}$
	4)5336·94
	1334·235
240	6671·175
240	6671·175
80	2223·725
8	222·3725
568	15788·4475 ÷ 365

	£15788·4475 × ·002
3	31·576895
10	10·52563
10	1·05256
	·10526
10000	43·26034
	·00432
	£43·25602
	or £43 5*s.* 1·44*d.* *Ans.*

*** The above question worked in the best style of the common method is here subjoined for comparison, all usual means of cancelling and simplification being taken advantage of.

$$\frac{£741\frac{29}{120} \times 568 \times 7\frac{1}{2}}{73000} = \frac{£88949 \times 568 \times 15}{73000 \times 120 \times 2}$$

$$= \frac{£88{\cdot}949 \times 71}{73 \times 2} = \frac{£44{\cdot}4745 \times 71}{73}$$

```
          £44·4745 × 71
          3113215
        ———————————
      73)3157·6895(£43·256
          292              20
          ———          ————————
           237          s. 5·120
           219               12
           ———          ————————
            186         d. 1·44
            146              Ans. £43 5s. 1·44d.
            ———
             408
             365
             ———
              439
              438
              ———
               15
```

Exercises 8.

Find the simple interest on

	£	s.	d.		days		p.c.
1.	114	15	0	for	83	at	5
2.	462	18	6	,,	113	,,	4
3.	129	11	8	,,	206	,,	3
4.	84	11	9	for	278	at	$3\frac{1}{2}$
5.	491	16	9	,,	223	,,	$2\frac{1}{2}$
6.	1108	13	9	,,	191	,,	$5\frac{1}{4}$

7. Calculate the simple interest on £1329 14*s.* 11*d.* from February 10 to December 3, 1883, at $4\frac{1}{2}$ per cent. per annum.

8. What is the mercantile discount on a bill for £94 17*s.* 4*d.*, payable on March 1, 1884, discounted on October 10, 1883, at $3\frac{1}{4}$ per cent. per annum?

CHAPTER VI.

SHORT METHODS IN THE CALCULATION OF COMPOUND INTEREST.

1. When questions in Compound Interest are included in Arithmetic Examination Papers, the number of years proposed is always small, rarely exceeding 5, and usually limited to 3 or 4. This is on account of the heavy amount of calculation requisite for longer periods.

For the requirements of business Compound Interest Tables are published. It is, however, necessary to the attainment of a thorough knowledge of Arithmetic that the principle involved in Compound Interest, and the process of computing it, should be intelligently known. And for this purpose the given period need not exceed four or five years.

But there is no Rule of Arithmetic of which the examples are more commonly worked in a tedious and confused manner than Compound Interest. We will here, therefore, exemplify some good methods of economising time and work and securing distinct arrangement in the working of this Rule.

Ex. 1. What would be the amount of £147 11*s*. 3*d*. in 5 years at 2 per cent. per annum, compound interest?

The given principal multiplied by $1{\cdot}02^5$ will produce the amount required. Now, to raise 1·02 to the 5th power is to find the amount of $1{\cdot}02 \times 1{\cdot}02 \times 1{\cdot}02 \times 1{\cdot}02 \times 1{\cdot}02$, which may be done as in the left-hand portion of the subjoined work, either dividing successively by 50, or else multiplying successively by ·02, for the interest to be added, and observing in the multiplication by ·02 to keep the first figure of the product two places to the right of the multiplicand.

Then we have to obtain the product of £147 11*s*. 3*d*. by 1·1040808, for the amount at the end of 5 years.

	1·02	× ·02	1·1040808
$\frac{2}{100} = \frac{1}{50} =$	·0204		147
	1·0404	× ·02	77285656
	·020808		154571312
	1·061208	× ·02	162·29988
	·0212242		10*s*. 0*d*. $= \frac{1}{2} =$ ·55204
	1·0824322	× ·02	1 3 $= \frac{1}{8} =$ ·06900
	·0216486		162·92092
$1{\cdot}02^5 =$	1·1040808		or £162 18*s*. 5·02*d*. *Ans.*

Otherwise and more conveniently :

Let the given principal have its value expressed in the denomination of pounds; then add successively $\frac{1}{50}$, or multiply successively by ·02, as in the form annexed, to obtain the successive amounts from year to year.

£147·5625	× ·02
2·95125	
150·51375	
3·01027	
153·52402	
3·07048	
156·59450	
3·13189	
159·72639	
3·19453	
£162·92092	*Ans.*

12	3·0*d.*
20	11·25*s.*

Given principal = £147·5625

2. When it is required to find the principal that will amount to a given sum in a given number of years at a given rate by compound interest, or, in other words, to find what would be the present worth of a given sum payable so many years hence, were compound interest allowed at a certain rate, we must employ the amount of 1 as found in the first of the preceding methods of operation. Thus, to find the principal that amounted to £162 18*s.* 5·02*d.* by compound interest in 5 years at 2 per cent. per annum, we have to divide £162·92092 by 1·02^{5}, that is, by 1·104081.

The principal could be traced back by successive divisions by 1·02; but the process would be too laborious.

3. Now, to involve the amount of 1 for a year to the power denoted by the number of years, it is a general practice with young students to proceed as in ordinary multiplication. But much greater ease and expedition are secured by the method we have exemplified in the involution of 1·02; and similarly, the involution for the rates 5, 4, 3, $2\frac{1}{2}$, $3\frac{1}{4}$, $4\frac{1}{2}$, should be begun as follows:—

	1·05
5 p.c. = $\frac{1}{20}$ =	·0525
	1·1025

		(*Otherwise.*)	
	1·04		
2 p.c. = $\frac{1}{50}$ =	·0208	1·04	× ·04
	·0208	·0416	
	1·0816	1·0816	

		(*Otherwise.*)	
	1·03		
2 p.c. = $\frac{1}{50}$ =	·0206	1:03	× ·03
1 p.c. = $\frac{1}{100}$ =	·0103	·0309	
	1·0609	1·0609	

	1·025
$2\frac{1}{2}$ = $\frac{1}{40}$ =	·025625
	1·050625

1·0325 × ·03	1·045 × ·04
·030975	·04180
$\frac{1}{4}$ p. c. = $\frac{1}{400}$ = ·0025812	$\frac{1}{2}$ p. c. = $\frac{1}{200}$ = ·005225
1·0660562	1·092025

Otherwise, $3\frac{1}{4}$ per cent. may be calculated by adding to 1·0325, the aliquot parts 2 p.c. $= \frac{1}{50}$, 1 p.c. $= \frac{1}{100}$, and $\frac{1}{4}$ of 1 p.c.; also $4\frac{1}{2}$ p.c. may be divided into $2\frac{1}{2} = \frac{1}{40}$ and $2 = \frac{1}{50}$. This way is preferable when the index of the power is high, because division ensures accuracy as far as we choose to carry the approximation.

The following worked examples may now be examined.

Ex. 2. Of what sum payable 3 years hence would £888 19*s.* $11\frac{1}{8}$*d.* be the present worth, if compound interest were allowed at 4 per cent. per annum?

The question is easily seen to be only an indirect way of asking the amount of the given sum in 3 years at 4 per cent. per annum. We might have divided the rate into two equal parts, each 2 p.c. $= \frac{1}{50}$.

12 | 11·125*d.*
20 | 19·927083*s.*
£888·996354 × ·04
35·559854
924·55621 × ·04
36·98225
961·53846 × ·04
38·46154
£1000·00000 *Ans.*

Ex. 3. What principal would in 4 years amount to £600 by compound interest at $7\frac{1}{2}$ per cent. per annum?

$1·075^4$ = 1·335469)600·000000
£449·28036

5 p.c. = $\frac{1}{20}$ = ·05375
$2\frac{1}{2}$ p.c. = $\frac{1}{2}$ = ·026875
1·155625
·0577812
·0288906
1·2422968
·0621148
·0310574
1·3354690

6581240
12393640
374419
107325
487
87
£449·28036
20
5·6072
12

Ans. £449 5*s.* 7·2864*d.*

Ex. 4. On what principal would £53 11*s.* 9*d.* be the compound interest for 5 years, at $4\frac{1}{3}$ per cent. per annum?

	$1·0433333^5$	= 1·236276
$3\frac{1}{3}$ p.c. = $\frac{1}{30}$ =	·0347777	1·0
1 p.c. = $\frac{1}{100}$ =	·0104333.	·236276 = int. on 1 ;
	1·0885444	
	·0362848	20)11·75*s.*
	·0108854	·236276)53·5875
	1·135715	£226·8
	·037857	47255
	·011357	6332
	1·184929	1606
	·039498	189
	·011819	
	1·236276	£226 8 = =£226 16*s.* *Ans.*

Exercises 9.

1. Find the amount by compound interest of £186 5*s.* in 4 years, at 3 per cent. per annum.

2. To what sum will £654 3*s.* 3*d.* amount by compound interest in 4 years, at $3\frac{1}{2}$ per cent. per annum?

3. Find the compound interest on £1179 14*s.* 7*d.* for 3 years, at $3\frac{1}{4}$ per cent. per annum.

4. What principal would amount to £400 by compound interest in 5 years, at $2\frac{1}{2}$ per cent. per annum?

5. What principal would amount to £1178 4*s.* 1*d.* in 3 years, by compound interest, at $5\frac{1}{4}$ per cent. per annum?

6. On what principal would 6 years' compound interest, at $4\frac{1}{2}$ per cent. per annum be £17 5*s.* 7*d.*

CHAPTER VII.

SHORT METHODS IN THE WORK OF AN EXAMINATION PAPER.

[*Set, under direction of the Civil Service Commissioners, at a competition for Men Clerkships, Lower Division, in May* 1883.]

QUESTIONS.

1. Express as a decimal the value of $\frac{1}{820}$ of the product of $6\cdot\dot{3}0769\dot{2}$ and $1\cdot\dot{4}2857\dot{1}$.

2. How many plots, each containing 7 sq. ft. 38 ins., can be formed out of a field of 8 poles 19 sq. yds. 4 ft. 72 ins.?

3. If 9 gallons of brandy, together with $1\frac{7}{20}$ cwt. of sugar, cost as much as 12 tons of coals, while $\frac{1}{3}$ gallon of brandy costs the same as $\frac{1}{5}$ cwt. of sugar, how much sugar is equal in value to 10 tons of coals?

4. Supposing a building in the form of a cube to contain 1259712 cubic feet, what would be the cost of carpeting its floor with carpet $\frac{3}{4}$ yard wide, worth 3*s.* $4\frac{1}{2}$*d.* per yard?

5. Determine by duodecimals the area of a rectangle whose adjacent sides respectively measure 10 ft. $9\frac{1}{3}$ ins. and 5 ft. $2\frac{1}{2}$ ins. How many such rectangles are there in one which contains 12125 sq. ft.?

6. If $\frac{4}{5}$ of the difference between a certain fraction and $\frac{2}{3}$ lie between $1\frac{1}{15}$ and $1\frac{1}{3}$, find between what limits the fraction itself must lie.

7. If it be known that the decimal ·2421 or some portion of it repeats, calculate as a vulgar fraction the difference between the greatest and least values which it can have.

8. Extract the square root of 25400544 in the scale of 6; transform 3065·263 from the scale of 8 to that of 10; and reduce $\frac{1}{48}$ from the scale of 10 to a duodecimal fraction.

9. Find what is the first year in which a sum of money will become more than doubled in amount, if put out at compound interest at the rate of 10 per cent. per annum.

10. Two clocks point to 5 P.M. at the same instant. One loses $7\frac{1}{2}$ seconds and the other gains $8\frac{1}{2}$ seconds in 24 hours. Find the interval which will elapse before one will be precisely $\frac{1}{2}$ an hour before the other, and the times which each will then indicate.

11. If to some wheat bought at 39*s.* per qr. some more is added costing 6*s.* per bushel, in what proportion must the better quality be mixed with the other, in order that, if the whole be sold at 57*s.* 6*d.* per quarter, the rate of profit may be 25 per cent.?

12. A. invests one-third of his money in the $2\frac{1}{2}$ per cents. at 90, the other two-thirds he lends to B. at 4 per cent. simple interest. B. pays the interest duly for 4 years; at the end of another 4 years he becomes bankrupt and pays a dividend of 13*s.* 4*d.* in the pound to A. on the principal and interest owing. At the same time A. sells out his stock at 80, and then finds that the whole sum received during the 8 years as principal and interest is £10 less than that with which he began. What was his original amount?

13. Divide £2259 10*s.* 7*d.* amongst four persons, giving to the second half as much again as to the first, to the third one-third less than to the first and second together, and to the fourth one-third less than to the second and third together.

14. If by delaying an investment in $2\frac{1}{2}$ per cent. stock while the price paid falls from 90 to $89\frac{3}{8}$, the annual income to be received is increased by £6 5*s.*, what is the sum invested?

15. A. sells gold lace costing £375 per piece to B. at £518 15*s.* per piece, and gives him 9 months credit. At the same time B. sells silk to A. which has cost him £125 per piece, and gives him 6 months credit.

If the two invoices are made out for the same amount, at what price per piece must B. invoice the silk, in order that, at the moment of sale, the total amount of A.'s profit may equal that of B.; the present values of the invoices being determined upon the supposition that the interest of money is 5 per cent. per annum?

Solutions.

1. $6\frac{307692}{999999} \times 1\frac{428571}{999999} = 6\frac{34188}{111111} \times 1\frac{47619}{111111}$

$$= 6\tfrac{4}{13} \times 1\tfrac{3}{7}\text{; hence } \tfrac{1}{820} \text{ of } \tfrac{82}{13} \times \tfrac{10}{7} = \frac{1}{7 \times 13}$$
$$= \cdot142857\tfrac{1}{7} \div 13 = \cdot\dot{1}4285\dot{7} \div 13$$
$$= \cdot\dot{0}1098\dot{9}. \quad \textit{Ans.}$$

Note.—A candidate would be required to show, on some space apart from the direct solution, the work of finding greatest common measures for the reduction of the two fractions on the right of the first line. We subjoin the work :—

Strike out 4)34188 ;	Strike out 7)111111
G. C. M. 8547)111111(13	G. C. M. 15873)47619(3

2. 8 sq. po. $19\frac{1}{2}$ yds. $\div$ $7\frac{19}{72}$ sq. ft.

$$= 261\tfrac{1}{2} \text{ sq. yds.} \div \frac{523}{72 \times 9} \text{ sq. yds.} = 523 \times \frac{36 \times 9}{523}.$$
$$= 36 \times 9 = 324 \text{ plots.} \quad \textit{Ans.}$$

3. Since 12 tons of C. = 9 gall. of B. + $\frac{27}{20}$ cwt. of S., dividing throughout by 27 shows that

$$\tfrac{4}{9} \text{ ton of C.} = \tfrac{1}{3} \text{ gall. of B.} + \tfrac{1}{20} \text{ cwt. of S.,}$$
$$= \tfrac{1}{5} + \tfrac{1}{20}, \text{ or } \tfrac{1}{4} \text{ cwt. of S.,}$$
$$\therefore 10 \text{ tons, or } \tfrac{4}{9} \times \tfrac{9}{4} \times 10 \text{ tons, of C.} = \tfrac{1}{4} \times \tfrac{9}{4} \times 10 \text{ cwt. of S.}$$
$$= 45 \text{ cwt.} \div 8 = 5\tfrac{5}{8} \text{ cwt. of S.} \quad \textit{Ans.}$$

4. $\sqrt[3]{1259712} = 108$, the number of feet for each side of the floor, = 36 yards;

$$\text{hence, length of carpet required} = 36 \text{ yds.} \times 36 \div \tfrac{3}{4}$$
$$= 144 \text{ yds.} \times 12 \text{ at } 40\tfrac{1}{2}d. = 144 \text{ at } 40\tfrac{1}{2}s.$$
$$= £4\ 1s. \times 72 = £291\ 12s. \quad \textit{Ans.}$$

Note.—We annex the work of extracting the cube root. It is easy to see that the first two figures of the root will not exceed 10, which we therefore write at once.

$$\begin{array}{r|l} 10^2 \times 300 & 1259712(108 \\ = 30000 & 1000 \\ 308 \times 8 = 2464 & \overline{259712} \\ \overline{32464} & 259712 \\ & \overline{} \end{array}$$

5.

10 ft.	9 in.	4′	
5	2	6	
53	10	8	
1	9	6	8
	5	4	8
56	1	$7\frac{1}{3}$	
	12		

56 sq. ft. $19\frac{1}{3}$ sq. in. = area. *Ans.*

$$12125 \div 56\tfrac{29}{216}$$
$$= \frac{12125 \times 12}{673\frac{11}{18}}$$
$$= \frac{12125 \times 12 \times 2}{1347\frac{2}{9}}$$
$$= 12 \times 2 \times 9 = 216 \ \textit{Ans.}$$

6. $\frac{4}{5}$ of the difference referred to is $> \frac{16}{15}$ and $< \frac{20}{15}$;
$\therefore$ the difference is $> \frac{16}{15} \div \frac{4}{5}$, that is, $> 1\frac{1}{3}$,
$\therefore$ the fraction itself is $> (1\frac{1}{3} + \frac{2}{3})$, or > 2;
but the difference is $< \frac{20}{15} \div \frac{4}{5}$, that is $< 1\frac{2}{3}$,
$\therefore$ the fraction itself is $< (1\frac{2}{3} + \frac{2}{3})$, or $< 2\frac{1}{3}$;
Hence, the fraction lies between 2 and $2\frac{1}{3}$. *Ans.*

7. The recurring form of highest value is $\cdot 242\dot{1}$, and that of lowest value is $\cdot 242\dot{1}$.

$$\begin{array}{rl} & \cdot 242124212421 + \&c. \\ & \cdot 242111111111 + \&c. \\ \text{difference,} & \cdot 000013101310 + \&c. = \cdot 000\dot{0}13\dot{1}, \\ & = \frac{131}{9999000}. \ \textit{Ans.} \end{array}$$

8. (i.) $\sqrt{25400544}$ senary = 4112 sen. *Ans.*

```
          24
121    |140
       |121
1221    1505
        1221
12222   24444
        24444
```

(ii.) The given quantity $= 3065\frac{263}{1000}$ octonary; and now we shall convert each of the three octonary numbers to the denary or common scale;

3065,	263,	1000;
8	8	8
24	22	8
8	8	8
198	179	64
8		8
1589		512

the given quantity, therefore, is equal to $1589\frac{179}{512}$ denary; and the fraction here may be reduced to the decimal form by dividing by $8 \times 8 \times 8$, successively, as under; then, prefixing the integral part to the result gives the answer.

$$\begin{array}{l} 8)179 \\ \hline 8)22{\cdot}375 \\ \hline 8)2{\cdot}796875 \\ \hline 1589{\cdot}349609375. \textit{ Ans.} \end{array}$$

(iii.) $\frac{1}{48}$ denary $= \frac{1}{40}$ duodenary. *Ans.*

Note.—The octonary radix fraction might be reduced to the required form of tenths, hundredths, thousandths, &c. by multiplying the fractional part successively by 10, observing to divide the product of each figure by 8 as we go along. We annex only a few steps of this process, inserting the multiplier only once. The figures preceding the point are those of the required decimal.[1]

$$\begin{array}{l} {\cdot}263 \text{ octonary} \\ \underline{\quad 10} \\ 3{\cdot}376 \\ 4{\cdot}754 \\ 9{\cdot}470 \\ 6{\cdot}060 \\ 0{\cdot}740, \text{ \&c.} \end{array}$$

9. Here we have to find what power of 1·1 first exceeds in amount the number 2; and this is here meant to be done by trial, the successive amounts of 1 at the end of each year being calculated as follows:—

$$\begin{array}{ll} \text{1st year,} & 1{\cdot}1 \\ \frac{1}{10} = & \underline{{\cdot}11} \\ \text{2nd ,,} & 1{\cdot}21 \\ & \underline{{\cdot}121} \\ \text{3rd ,,} & 1{\cdot}331 \\ & \underline{{\cdot}133} \\ \text{4th ,,} & 1{\cdot}464 \\ & \underline{{\cdot}146} \\ \text{5th ,,} & 1{\cdot}610 \end{array} \qquad \begin{array}{ll} & 1{\cdot}610 \\ \frac{1}{10} = & \underline{{\cdot}161} \\ \text{6th year,} & 1{\cdot}771 \\ & \underline{{\cdot}177} \\ \text{7th ,,} & 1{\cdot}948 \\ & \underline{{\cdot}195} \\ \text{8th ,,} & 2{\cdot}143 \end{array}$$

Ans. In the 8th year.

Note.—The general form of solution for this problem is that which determines the value of x so that $1{\cdot}1^x$ may

[1] *See* the author's 'Supplementary Arithmetic,' chap. vi. § 5.

be equal to a given quantity. The solution is worked by means of logarithms.

Here we have $1{\cdot}1^x = 2+$, to find x.

$$\text{Now } x = \frac{\log\ 2+}{\log\ 1{\cdot}1} = \frac{\log\ 2+}{\log\ 11-1} = \frac{30103+}{4139}$$

= more than 7. Hence, in the 8th year. *Ans.*

10. The differences of time indicated will be 16 seconds the 1st day, 32 seconds the 2nd day, 48 seconds the 3rd day, and so on; and this series will grow to 30 minutes, or 1800 seconds, in $1800 \div 16 = 112\frac{1}{2}$ days of 24 hours: which is the first *Answer*.

Again: At the end of 225 *half-days* the slower clock will indicate $3\frac{3}{4}$ seconds $\times$ 225 before 5 A.M. $=(675+168\frac{3}{4})$ sec. $=843\frac{3}{4}$ sec. $=14$ min. $3\frac{3}{4}$ sec. to 5 A.M.; and, therefore, the faster clock will indicate 30 min. $-$ 14 min. $3\frac{3}{4}$ sec., or 15 min. $56\frac{1}{4}$ sec. past 5 A.M.: which are the other *Answers*.

11. Prime cost of the mixture$=\frac{100}{125}$ of $57\frac{1}{2}s.=230s. \div 5=46s.$

The two qualities cost 39*s.* and 48*s.* per quarter, respectively; and these rates are, respectively, 7*s.* *below* and 2*s.* *above* the average rate; therefore the proportions mixed must have been 7 of the better quality to 2 of the inferior. *Ans.*

12. For every £90 that he invests in stock he lends £180; together$=$£270.

Now, for the £90 invested he gets £$2\frac{1}{2} \times 8, =$ £20 of dividends, and £80 by selling out, which are together $=$£100.

For the loan of £180 he gets

$$\begin{aligned} &\tfrac{16}{100} \text{ of } £180 + \tfrac{2}{3} \text{ of } 1\tfrac{16}{100} \text{ of } £180, \\ &= £28{\cdot}8 + \tfrac{116}{100} \text{ of } £120, \\ &= £28{\cdot}8 + £139{\cdot}2 = £168; \end{aligned}$$

his total returns, therefore, are £100 + £168 = £268 for an investment of £270; and the question now is, if on £270 he loses £2, on what sum does he lose £10?

$$£270 \times 5 = £1350. \ \textit{Ans.}$$

13. The shares are as 1, $1\frac{1}{2}$, $1\frac{2}{3}$, and $2\frac{1}{9}$, or as 18, 27, 30, and 38. Hence the first person will have $\frac{18}{113}$ of £2259 10*s.* 7*d*, the second $\frac{27}{113}$ of that sum, the third $\frac{30}{113}$, and the fourth $\frac{38}{113}$.

```
113)£2259 10s. 7d.(          £19 19s. 11d.
    113                                3
    ----                     -------------
    1129                      £59 19   9 = 3/113
    1017                      ------------
    ----                      359 18   6 = 18/113
     112                      539 17   9 = 27/113
    ----                      599 17   6 = 30/113
    2250(19                  -------------
    113                      1499 13   9
    ----                     2259 10   7
    1120                     -------------
    1017                      759 16  10
    ----                     -------------
     103
    ----
    1243(11
    1243
    ----
```

Hence the shares are £359 18*s.* 6*d.*
" " 539 17 9
" " 599 17 6
" " 759 16 10 } *Ans.*

14. The rates of income per pound of investment are, respectively, $\frac{£2\frac{1}{2}}{90}$ and $\frac{£2\frac{1}{2}}{89\frac{3}{8}}$ or $£\frac{1}{36}$ and $£\frac{4}{143}$; hence $(\frac{4}{143}-\frac{1}{36})$ of the no. of pounds invested $=£6\frac{1}{4}$ or, multiplying throughout by 36, we have

$$(\tfrac{144}{143}-1) \text{ of the sum invested} = £225;$$
$$\therefore £225 \div \tfrac{1}{143} = £1287 \times 25 = £32175. \quad \textit{Ans.}$$

15. Let us deal with one piece as given by A., and find the corresponding quantity given by B.

The amount of A.'s invoice is $£518\frac{3}{4}$; that of B.'s is the same; but *at the moment of sale* the present worth of A.'s invoice is that of $£518\frac{3}{4}$ for 9 months at 5 per cent. per annum, viz. $\frac{100}{103\frac{3}{4}}$, or $\frac{80}{83}$, of $£\frac{2075}{4}$

$$= \tfrac{20}{83} \text{ of } £\tfrac{8300}{4} = £500;$$

and the present worth of B.'s invoice at the same mo-

ment is that of £$518\frac{3}{4}$ for 6 months, viz. $\frac{100}{102\frac{1}{2}}$, or $\frac{40}{41}$ of £$\frac{2075}{4}$

$$=\tfrac{10}{41} \text{ of } £2075 = £\tfrac{20750}{41};$$

A.'s *present* gain is £500 *minus* £375 = £125; B.'s *present* gain is also £125, which must be equal to £$\frac{20750}{41}$ *minus* whatever the proportionate quantity of silk, at £125 per piece, cost him; hence £$\frac{20750}{41}$ − £125 is the prime cost of B.'s no. of pieces at £125 per piece; so that B.'s proportionate quantity is equal to

$$(£\tfrac{20750}{41} - £125) \div £125 = \tfrac{166}{41} - 1 = \tfrac{125}{41} \text{ pieces};$$

and therefore he invoiced the silk at £$518\frac{3}{4} \div \frac{125}{41}$

$$= \frac{£518\frac{3}{4} \times 8 \times 41}{1000} = £4{\cdot}15 \times 41 = £170\ 3s.\ \textit{Ans.}$$

CHAPTER VIII.

MISCELLANEOUS QUESTIONS.

1. Find the highest common factor in 4389 and 4218.

2. What would be the cost of 19 tons at £1 7*s.* $10\frac{1}{2}$*d.* per cwt.?

3. Find the cost of 23 cwt. at £6 11*s.* $8\frac{1}{2}$*d.* per ton.

4. Calculate 75 times 239, and the 75th part of 1426054.

5. Multiply 6834 by 5397, and 1368 by 99.

6. Calculate 460 things at £1 14*s.* 9*d.*, and 365 at £2 15*s.* $4\frac{1}{2}$*d.*

7. Multiply ·01236 by 6·321, and 4567 by 125.

8. Find the greatest common measure of 39780 and 370188.

9. Calculate 289 cwt. at 16*s.* $10\frac{1}{2}$*d.*, and $193\frac{1}{4}$ cwt. at 12*s.* $3\frac{3}{4}$*d.*

10. Multiply 732·84 by 4789, and 578 by 75.

11. A person's yearly income is £225 13*s.*: how much is that per day?

12. Calculate 365 days at 9*s.* $11\frac{1}{2}$*d.* per day.

13. Find the G.C.M. of 15987 and 13505.

14. Find the amount of 25 times 9999 × 99, and that of 999 times 674 × 75.

15. Calculate 310 lbs. at 8*s.* 5$\frac{1}{4}$*d.*, and 477 at 6*s.* 1$\frac{3}{4}$*d.*

16. Divide the product of 8526 and 3794 by 125.

17. What would an allowance of 4*s.* 5$\frac{1}{2}$*d.* a day amount to in the year 1884?

18. Divide the square of 109·26 by 365.

19. Reduce 27365 gallons to cubic inches.

20. Calculate 47 yards at 20$\frac{3}{4}$*d.*, and 53 at 19$\frac{1}{8}$*d.*

21. Find the continued product of 2968, 2869, and 7926, and the quotient of 7792 divided by 75.

22. Calculate $\frac{1}{75}$ of $\frac{1}{365}$ of 23791.

23. Find the G.C.M. of 21669 and 27028.

24. The product of two numbers is 294426, and the square of one of them is 9801: Find the other.

25. Calculate 87 things at 13*s.* 2$\frac{1}{4}$*d.*

26. Divide 23706$\frac{1}{6}$ by the 6th part of 365.

27. Find the simple interest on £217 14*s.* 10*d.* for 230 days, at 6$\frac{3}{4}$ per cent. per annum.

28. What is the amount of 93 times £3 6*s.* 9$\frac{3}{4}$*d.*, and that of 511 times 37*s.* 8*d.*?

29. Multiply 4 times 123456789 by $\frac{1}{9}$th of 625.

30. Work out 147321 × 123741, and 119$\frac{1}{4}$ × 25.

31. When £15 16*s.* 9$\frac{1}{2}$*d.* is the interest on £396 for a year, how much is a year's interest on £428?

32. What is the highest common factor in 9729, 13113, and 113646?

33. Find the simple interest on £1385 15*s.* 11*d.* for 118 days at 2$\frac{1}{2}$ per cent. per annum.

34. Calculate 3 years' compound interest on £364 15*s.* 6*d.* at 4 per cent. per annum.

35. The volume of a sphere is ·5236 of the cube of its diameter: Find the volume of a sphere of which the diameter is 7·32 feet.

36. Find the amount of 215 yards at 2*s.* 5$\frac{7}{8}$*d.*, and that of 117$\frac{5}{8}$ stone at 7*s.* 6*d.*

37. A rectangular cistern is 11 feet long and 7 feet wide, and its capacity is 3165 gallons: What is its depth?

38. Find the simple interest on £74 3*s.* 10$\frac{1}{2}$*d.* for 1 year 179 days at 3 per cent. per annum.

39. What is the value of 117$\frac{1}{2}$ cwt. of cheese, at the rate of £391 for 99 cwt.?

40. Calculate 504 things at 7*s.* 8$\frac{1}{2}$*d.*, and 154 at 13*s.* 6$\frac{3}{4}$*d.*

41. Find the greatest common measure of 92463, 156604, and 131495.

42. Calculate 182$\frac{1}{4}$ yards at 11$\frac{1}{4}$*d.*, and 76 at 1*s.* 10$\frac{3}{8}$*d.*

43. Find the amount of £1236 18*s.* by compound interest for 4 years, at 5 per cent. per annum.

44. If 163 ounces of tea be worth as much as 365 ounces of coffee, find, in ounces and the decimal of an ounce, what quantity of tea is worth as much as 217 ounces of coffee?

45. Reduce to a decimal, accurate to five places—

$$\frac{1}{2.5} + \frac{1}{2.5.8} + \frac{1}{2.5.8.11} + \text{\&c.}$$ [1]

46. Divide the product of 365 and 563 by 99.

47. Find the simple interest on £415 17*s.* 3*d.* for 343 days at 2$\frac{1}{4}$ per cent. per annum.

48. Calculate 365 lbs. at 7$\frac{5}{8}$*d.*, and 149$\frac{1}{2}$ yards at 9$\frac{5}{12}$*d.*

49. Convert 89102 cubic inches to gallons.

50. A reservoir holds 5617 gallons of water: What is the weight of the water in tons, at the rate of 1000 ounces per cubic foot?

51. What will £159 4*s.* 7*d.* amount to, by compound interest at 6 per cent. per annum, in 3 years?

52. The produce of a field of 11·27 acres being 365 bushels of grain, what extent of it yields 42 quarters?

53. Find the G.C.M. of 2002, 3198, and 4745.

54. Simplify $\frac{125}{121}$ of $1{\cdot}\dot{0}012\dot{3}$ of $4768{\cdot}\dot{4}0\dot{6}$.

55. How many gallons of water will weigh a ton, if a cubic foot of water weigh 1000 ounces?

56. From 365 times 56493 subtract $\frac{1}{99}$th of 394650.

[1] Points placed like a full stop between figures denote multiplication.

57. Calculate $63\frac{1}{4}$ yards at $19\frac{1}{4}d.$ per dozen yards, and $35\frac{3}{4}$ yards at $17\frac{7}{8}d.$ per dozen yards.

58. Find the amount of £93 7*s.* 5*d.* by compound interest in 4 years at $3\frac{1}{2}$ per cent. per annum.

59. Find the simple interest on £251 5*s.*—from June 15, 1883 to August 27, 1884—at $3\frac{1}{3}$ per cent. per annum.

60. A rectangular field is 896 links long and 479 broad: How many acres (each 10000 square links) does it contain?

61. Find a 4th proportional to 75, 381·52, and 7·385.

62. What is the value of $\frac{364}{365}$ of £48. 6*s.*?

63. When $36\frac{1}{2}$ yards of silk are worth £9 14*s.* $2\frac{1}{4}d.$, what is the value of 276 yards?

64. Calculate $137\frac{3}{4}$ ounces of gold at £3 17*s.* $10\frac{1}{2}d.$, and $52\frac{1}{2}$ ounces of silver at 4*s.* $10\frac{1}{4}d.$

65. What is the compound interest on £1056 11*s.* 2*d.*, for 3 years, at $3\frac{1}{4}$ per cent. per annum?

66. Calculate $214\frac{1}{2}$ yards at 3*s.* $4\frac{7}{12}d.$ per dozen yards, and 147 yards at $5\frac{3}{8}d.$ per dozen yards.

67. A cubical cistern can hold 4653 gallons: Find its depth in feet.

68. The interest on a certain sum of money in 225 days was £30 16*s.* $5\frac{1}{4}d.$: How much interest did it bear in 193 days?

69. What principal would make £13 10*s.* more by compound than by simple interest in 2 years at 5 per cent. per annum?

70. Multiply the double of 163853 by $49\frac{49}{99}$.

71. Calculate $26\frac{3}{8}$ yards at $11\frac{11}{12}d.$, and $71\frac{3}{4}$ at 1*s.* $5\frac{3}{4}d.$

72. The diameter of a cylindrical tank is 27 feet 7 inches, and its depth 15 feet 11 inches: How many gallons of water can it contain? [Area of a circle = ·7854 of that of its circumscribing square.]

73. A bookseller buys 68 copies of a book published at 6*s.* 6*d.* He is allowed 25 per cent. discount on the publishing price, and is charged for the 68 copies the net cost of 63. At what rate per cent. of discount on the publishing price can he retail the book, and make a total profit of £1 12*s.* $10\frac{1}{2}d.$?

74. Calculate to five decimal places the value of—

$$2 + \frac{2}{3} + \frac{2}{3.5} + \frac{2}{3.5.7} + \frac{2}{3.5.7.9} + \text{\&c.}$$

75. When the base of an isosceles right-angled triangle is 99, each of the equal sides is almost exactly 70: Hence find each of the equal sides in a similar triangle whose base is 23·456.

76. Suppose a piece of metal 5·12 inches long, 3·75 inches wide and 2·43 inches thick, to be melted, without loss, into a cube: What would be the length of the edge of the cube? and what difference would be made in the amount of surface?

77. When 1595 yards of calico cost as much as 427 yards of linen, and 1308 yards of flannel as much as 1025 yards of linen, find, to two places of decimals, the quantity of calico that costs as much as 365 yards of flannel.

78. A rectangular tank is 32 feet long and 18 feet 8 inches wide: What will be the depth of water in the tank when the weight of the water, at $62\frac{1}{2}$ lb. per cubic foot, is 56 tons 5 cwt.?

79. Suppose the rate of a clock to be $\cdot 0\dot{5}$ per cent. too fast, and to continue at that rate from July 1 to December 31; how much time would it gain in the half-year?

80. A gas-meter, after being used for 96 days, registers 11280 cubic feet as the consumption. The meter is then tested, and found to register $17\frac{1}{2}$ per cent. in excess of the gas actually consumed. Find the amount that ought to be charged, at 3*s.* 9*d.* per thousand cubic feet.

81. How many cubic inches of copper are equal in weight to 963 cubic inches of iron, if 4214 cubic inches of copper weigh as much as 3257 of lead, and 1460 of iron as much as 986 of lead?

82. When the side of an equilateral triangle is 40 inches, its area is 693 square inches; hence find the side of an equilateral triangle of which the area is 1171·24 square inches; the areas of similar triangles being as the squares of their corresponding sides.

83. The floor of a rectangular room, 16 feet high, is twice as long as it is wide, and requires $60\frac{1}{2}$ square yards of carpet to cover it: What will the painting of its walls come to at 3*s.* $5\frac{1}{2}$*d.* per square yard, covering allowance for windows and fireplace?

84. If I invest my money in Dock shares paying £7 per share, when the £100 share is at $122\frac{1}{2}$, I find that I get £34 10*s.* 6*d.* a year more than if I invest in $5\frac{1}{4}$ India Bonds at 105: What is my capital?

85. At what price per gallon must I invoice my sales of brandy that cost me 17*s.* 4*d.* a gallon that I may have $12\frac{1}{2}$ per cent. profit after allowing $2\frac{1}{2}$ per cent. discount?

86. The population of a country increases 3 per cent. annually, but is lessened annually by emigration to the extent of $\frac{1}{2}$ per cent. on the whole: What will be the increase per cent. at the end of three years?

87. When each of the equal sides of an isosceles right-angled triangle is 99, what is its altitude?

88. Reduce the nonary fraction $\frac{10201}{162150}$ to denary terms, then to lowest denary terms, and transform the latter result to express lowest nonary terms.

89. The three sides of a triangle are AB 25, BC 29, and AC 36: Find its area, and the length of BD, a perpendicular on AC.

90. The product of three numbers is $3057\frac{183}{256}$; the greatest of them is $17\frac{3}{4}$; and of the others the greater is not more than $3\frac{1}{16}$ of the other, nor less than $2\frac{1}{4}$ of it: Find the limits within which the least must lie.

91. I buy cloth at 13*s.* 6*d.* a yard, and am allowed 3 months' credit; and I sell it immediately at 14*s.* 8*d.*, allowing a certain term of credit, and find that, rating the interest of money at 5 per cent. per annum, the value of my gain at the moment of sale is $9\frac{1}{11}$ per cent. How long credit do I give?

92. With the earth excavated to form a uniform trench, $53\frac{1}{2}$ feet long and $12\frac{1}{2}$ feet wide, an embankment is made of 5136 cubic feet. Supposing the earth to have been increased $6\frac{1}{4}$ per cent. in bulk by the removal, what is the depth of the trench?

93. A mixture of 25 gallons of spirits costs me

£26 11*s.* 3*d.*, and is a blend of two qualities, for one of which I paid 25*s.* 3*d.* a gallon, and for the other 19*s.* a gallon : How many gallons were there of the better quality ?

94. Two unequal circles touch each other, the area of the larger being thrice that of the smaller : If the distance between their centres be 34·15 inches, what is the diameter of each ?

95. Extract the square root of $2523{\cdot}\dot{4}32\dot{3}$ in the scale of *seven*, working throughout according to septenary notation, and represent the fractional part of the answer as a septenary radix fraction.

96. If 27132 be the area of a rectangle of which the diagonal is 365, what is the area of a similar rectangle of which the diagonal is 132 ?

97. An alloy of silver is mixed with an alloy of gold in the proportion of 11·4 to 2·6. The percentage of dross in the silver alloy is 13·5, and in the gold 17·35 : Find the percentage of dross in the mixture.

98. A cistern 5 feet long and 3 feet 6 inches wide, filled with water, loses ·0005 of its depth of water by evaporation, the loss being found to amount to a quart of water : What is the depth of the cistern ?

99. A. and B. together can perform a piece of work in 77 hours : In what time could each do it alone if A. could do B.'s hourly amount of work in ·9604 of the time in which B. could do A.'s hourly amount of work ?

100. A. lends B. a certain sum ; at the same time he insures B.'s life for £737 12*s.* 6*d.*, paying annual premiums of £20. At the end of three years, and just before the fourth premium is to be paid, B. dies, having never repaid anything : What must A. have lent B. in order that he may just have enough to recoup himself, together with 5 per cent. compound interest on the sum lent and on the premiums ?

CHAPTER IX.

APPENDIX OF ADDITIONAL PROBLEMS IN HIGHER ARITHMETIC, WITH SOLUTIONS.

PROBLEMS.

1. The square root of a vulgar fraction is equal to ·7231, *very nearly*, its denominator being 153: Find its numerator.

2. Find three fractions of which the sum is $\frac{76}{81}$, the first being equal to 72 per cent. of the second, and the second equal to $\frac{3}{7}$ of the third.

3. A. bought property for £1041 13*s*. 4*d*., and sold it to B. at a certain rate of profit; B. sold it to C. at the same rate of profit, and C. sold it to D. for £1105 8*s*. 6*d*., gaining still at the same rate: What was that rate?

4. The sum of the circumferences of two circles touching each other is 99 inches: What is the distance between their centres? [The diameter of a circle is ·31831 of the circumference.]

5. Reduce to an undenary improper fraction, in lowest terms, the undenary expression $13{\cdot}\dot{8}641\dot{t}$, working throughout according to undenary notation. [*t* is used as a numeral for *ten*.]

6. Three trees, A, B, C, stand on a horizontal plane, C being exactly east from A, and south from B; moreover, C's distance from B is 37·86 yards, and A is 3·65 yards farther from B than from C: Find the distance from A to C.

7. If the areas of the squares described on the three sides of a triangle be 5, 9, and 20 square inches, what must be the *exact* area of the triangle?

8. Three bricklayers, A., B., C., build a wall. The whole would be built by A. alone in 14 days, or by B. in 18 days, or by C. in 21 days. They all begin together, and A. and B. continue till the wall is finished; but C.

leaves off a day before the completion of the work: How many days does the work last?

9. There are two cubical water tanks, the larger of which can hold 3192 gallons. The supply pipe of the smaller delivers $27\frac{1}{2}$ gallons of water per minute, that of the larger 70 gallons per minute. If the tanks are empty, and the supply pipes are set open together, both tanks will be filled at the same time: What must the depth of the smaller tank be if 11 gallons of water occupy 3050 cubic inches?

10. A certain amount of digging is done by A. and B. together in 4 days: What time would each take to do the whole work by himself, if the time that A. would require, in order to do as much as B. does in a day, is to the time that B. would require, in order to do as much as A. does in a day, as 95·663 is to 99?

11. A piece of metal weighing 12 cwt. 66 lbs. has been formed by compounding three metals in quantities which, by measure, are as 5 : 3 : 2; but the weights of equal volumes of them would be as 7 : 11 : 13. What weight of each of the component metals has been used?

12. A merchant, having bought 400 tons of coal, reckons that by selling them at 17*s.* $6\frac{1}{2}$*d.* per ton he will make $5\frac{1}{4}$ per cent. on his outlay; after selling 300 tons at that rate, he disposes of the remainder at a price which reduces his profit on the whole to 5 per cent. Find (1) what he gave for the 400 tons of coal, and (2) at what price per ton the second lot was sold.

13. A vessel making for a harbour fires a signal gun; the flash is seen from the harbour, and the sound follows in $22\frac{1}{2}$ seconds; a tug puts off immediately and steams in a straight course towards the vessel at the rate of 12 miles an hour; and from the tug, five minutes afterwards, the flash of a second gun is seen, the sound of which follows in 15 seconds. If sound travels 13 miles per minute, at what rate is the vessel approaching the harbour, and how soon after starting will the tug meet her?

SOLUTIONS.

1. $\cdot 7231^2 =$ the fraction $=$ numerator $\div$ 153 ;

$$\begin{array}{rl} \cdot 7231 & \\ \hline 50617 & \times 3 \\ 151851 & \\ 151851 & \\ \hline \cdot 52287361 & \end{array} \qquad\qquad \begin{array}{rl} \cdot 52287 & \\ 153 & \\ \hline 156861 & \times 5 \\ 784305 & \\ \hline 79 \cdot 99911 & = 80. \ \textit{Ans.} \end{array}$$

2. The *1st* is to the *2nd* as 72 is to 100 ;
the *2nd* is to the *3rd* as 100 is to $100 \div \frac{3}{7}$;
$\therefore$ the fractions are as 72, 100, $233\frac{1}{3}$,
or as 216, 300, 700, or as 54, 75, 175 ;
and the sum of these proportional parts is 304 ;
now $\frac{1}{304}$ of $\frac{76}{81} = \frac{1}{324}$; which severally multiplied by 54, 75, 175, gives $\frac{1}{6}, \frac{25}{108}, \frac{175}{324}$. *Ans.*

3. £$1041\frac{2}{3}$ multiplied by $R^3 =$ £$1105 \cdot 425$;

$$\frac{1105 \cdot 425 \times 12}{1041\frac{2}{3} \times 12} = \frac{132651}{125000} = \frac{1061208}{1000000};$$

$$\sqrt[3]{1 \cdot 061208} = 1 \cdot 02 = R \text{ ; hence 2 per cent. } \textit{Ans.}$$

4. The sum of the diameters is $\cdot 31831$ of the sum of the circumferences; $\therefore$ sum of the radii, or distance between centres

$$\begin{array}{rl} = \frac{1}{2} \text{ of } 99 \times & \cdot 3183100 \\ & 31831 \\ \hline = \frac{1}{2} \text{ of } & 31 \cdot 51269 \text{ in.} = 15 \cdot 756 \text{ in. } \textit{Ans.} \end{array}$$

5. The expression is $= 13 \dfrac{8641t}{ttttt}$ undenary ; and now we can find the G.C.M. of numerator and denominator to be 2469 undenary, as worked out below :—

$$\begin{array}{r|l|l} 8641t & ttttt & 1 \\ & 8641 & \\ \hline \end{array} \qquad\qquad \text{G.C.M.} = 2469 \ \begin{array}{|l|l} 8641t & 36 \\ 7295 & \\ \hline 1357t & \\ 1357t & \\ \hline \end{array}$$

24690, from which strike out 10, as not being a factor in 8641*t*.

The G.C.M. thus found reduces the expression to $13\frac{36}{46}$ undenary $= \frac{613}{46}$ undenary. *Ans.*

6. The distances between the trees form a right-angled triangle, in which AB is the hypotenuse.

BC^2 = the difference of the squares on AB and AC = the product of the sum and difference of AB and AC = $(AB + AC) \times 3{\cdot}65$; $\therefore$ $BC^2 \div 3{\cdot}65$ = the sum of AB and AC.

37·86		143338 × ·002
37·86	3	286·676
22716 × 3	10	95·558
68148 × 2	10	9·556
136296		·956
1433·3796	10000	392·746
		·040
or 1433·38 = BC^2	AB + AC =	392·706
1433·38 ÷ 3·65	AB − AC =	3·650
= 143338 ÷ 365		2)389·056 = 2 AC
	Ans. AC =	194·528 yds.

7. The three sides are equal to $\sqrt{5}$, 3, and $2\sqrt{5}$; and the half-sum of the sides is equal to $1\frac{1}{2}\sqrt{5} + 1\frac{1}{2}$; but to avoid fractions, let us first find the area of a triangle *four* times as large as the one given ; then, the sides are to be taken *twice* as great as those given, viz. $2\sqrt{5}$, 6, and $4\sqrt{5}$; the half-sum is now $3\sqrt{5} + 3$; and subtracting from this the three sides severally, and proceeding according to the well-known Rule, we have the quadruple area

$$= \sqrt{\left\{(3+\sqrt{5})(3-\sqrt{5})(3\sqrt{5}-3)(3\sqrt{5}+3\right\}} \;;$$

and here recognising, as in the preceding solution, that the sum of two quantities multiplied by their difference produces the difference of the squares of the quantities, the expression for the quadruple area becomes

$$\sqrt{(9-5)(45-9)} = \sqrt{4 \times 36} = 12\,;$$

hence the area of the given triangle is exactly 3. *Ans.*

8. Suppose the whole work to consist of 126 equal measures, that number being an exact common multiple of 14, 18, and 21 ; A. did 9 of these measures per day, B. 7, C. 6. Now, if C. had not left off a day before the completion of the 126 measures, he would have added six extra measures, making 132 measures to have been done

in the required time at the rate of $9+7+6$ or 22 measures per day; hence $132 \div 22 = 6$ days. *Ans.*

9. The smaller tank holds $\frac{27\frac{1}{2}}{70}$ or $\frac{11}{28}$ of 3192 gallons

$= \frac{3192}{28}$ of 11 gallons $= 114$ times 3050 cub. inches
$= 347700$ cub. in, the cube root of which is 70·318 in.
$= 5·86$ feet. *Ans.*

10. $95·663 \div 99$, or ·966293 of the time in which B. can do A.'s daily amount is equal to the time in which A. can do B.'s daily amount.

$$\begin{array}{r} = ·95663 \\ + ·0095663 \\ + ·0000956 \\ + ·0000010 \\ \hline = ·966293 \end{array}$$

Now, if A.'s daily amount of work be called a measures, and B.'s be called b measures, B. could do A.'s daily amount in $\frac{a}{b}$ of a day, and A. could do B.'s daily amount in $\frac{b}{a}$ of a day; hence $\frac{b}{a} = ·966293$ of $\frac{a}{b}$

$$\therefore ·966293 = \left(\frac{b}{a}\right)^2; \therefore \frac{b}{a} = \sqrt{·966293} = ·983;$$

accordingly, A.'s daily amount of work is to B.'s as 1000 : 983; and as they jointly do $\frac{1}{4}$ of the whole per day,

A. does $\frac{1000}{1983}$ of $\frac{1}{4}$ and B. $\frac{983}{1983}$ of $\frac{1}{4}$ per day;

$$\left.\begin{array}{l} \therefore \text{A. would do the whole in } \frac{1983 \times 4}{1000} = 7·932 \text{ days,} \\ \text{B} \quad \text{,,} \quad \text{,,} \quad \frac{1983 \times 4}{983} = 8·069 \text{ days.} \end{array}\right\} \textit{Ans.}$$

11. There were 5 units of volume each 7 units of weight,
3 ,, ,, 11 ,, ,,
and 2 ,, ,, 13 ,, ,,
$= 35 + 33 + 26 = 94$ units of weight;
hence, $\frac{35}{94}$ of 1410 lbs. $= 15$ lbs. $\times 35 = 525$ lbs.
$\frac{33}{94}$,, $= 15$ lbs. $\times 33 = 495$ lbs.
$\frac{26}{94}$,, $= 15$ lbs. $\times 26 = 390$ lbs.;
or the weights used were 4 cwt. 77 lbs., 4 cwt. 47 lbs. and 3 cwt. 54 lbs. *Ans.*

12. (i.) Gave for the whole $\frac{100}{105\frac{1}{4}}$ of $17s.\ 6\frac{1}{2}d. \times 400$ $= \frac{400}{421}$ of £$\frac{421}{24} \times 20 = \frac{1}{3}$ of £1000 $=$ £$333\frac{1}{3}$. *Ans.*

(ii.)

	£	s.	d.
Bought 400 tons for	£333	6s.	8d.
Gain on the whole $\frac{1}{20}$ =	16	13	4
Sold 400 tons for	350	0	0
300 at $17s.\ 6\frac{1}{2}d.$	= 263	2	6
∴ 100 sold for	£ 86	17	6

which is $17s.\ 4\frac{1}{2}d.$ per ton. *Ans.*

13. Sound being supposed to travel $\frac{13}{60}$ of a mile per second, the tug is at first $\frac{13}{60}$ miles $\times \frac{45}{2} = \frac{39}{8}$ miles, or $4\frac{7}{8}$ miles from the vessel.

The tug sails 1 mile in 5 minutes, and is then $\frac{13}{60}$ miles $\times 15 = 3\frac{1}{4}$ miles from the vessel; therefore the vessel sailed $4\frac{7}{8} - 3\frac{1}{4}$ or $1\frac{5}{8}$ miles in 5 minutes, which is at the rate of $19\frac{1}{2}$ miles an hour. *Ans.*

Secondly, at starting there were $4\frac{7}{8}$ miles to be sailed over between the vessel and the tug, at the respective rates of $19\frac{1}{2}$ and 12 miles an hour, $31\frac{1}{2}$ miles an hour together; therefore the tug would meet the vessel in $\frac{4\frac{7}{8}}{31\frac{1}{2}}$ or $\frac{13}{84}$ of an hour, $= 65$ min. $\div 7 = 9\frac{2}{7}$ min. *Ans.*

ANSWERS TO THE EXERCISES.

Exercises 1.

1. 93. 2. 92. 3. 199. 4. 95. 5. 365.
6. 73. 7. 41. 8. 43. 9. 67.

Exercises 2.

1. $\frac{63}{149}$. 2. $\frac{12}{23}$. 3. $\frac{19}{25}$. 4. $\frac{13}{77}$. 5. $\frac{35}{73}$.
6. $\frac{54}{59}$. 7. $\frac{31}{42}$. 8. $\frac{28}{151}$. 9. $\frac{17}{71}$. 10. $\frac{43}{67}$.
11. $\frac{173}{349}$. 12. $\frac{23}{127}$.

Exercises 3.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	6	11	$7\frac{1}{2}$	9.	18	8	3	17.	5	3	9
2.	74	19	$9\frac{3}{4}$	10.	11	11	$3\frac{1}{2}$	18.	13	14	$3\frac{3}{4}$
3.	89	18	$1\frac{1}{2}$	11.	9	7	9	19.	60	11	$7\frac{1}{2}$
4.	15	0	$8\frac{1}{4}$	12.	54	8	$0\frac{3}{4}$	20.	14	5	1
5.	6	8	4	13.	30	10	0	21.	41	10	$4\frac{3}{4}$
6.	8	2	6	14.	113	18	0	22.	102	7	6
7.	106	17	5	15.	87	1	$5\frac{1}{2}$				
8.	17	14	$4\frac{1}{2}$	16.	14	15	0				

Exercises 4.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	197	3	4	15.	431	0	$10\frac{1}{2}$	29.	3692	10	$7\frac{1}{2}$
2.	120	9	0	16.	2067	18	0	30.	176	11	$11\frac{1}{4}$
3.	79	0	0	17.	178	12	8	31.	354	19	$7\frac{1}{4}$
4.	167	15	1	18.	19	0	$2\frac{1}{4}$	32.	1712	11	9
5.	279	17	2	19.	22	17	$11\frac{3}{4}$	33.	718	13	$1\frac{1}{2}$
6.	483	1	8	20.	237	2	$2\frac{1}{4}$	34.	1832	2	0
7.	201	14	8	21.	144	3	0	35.	545	19	$3\frac{1}{2}$
8.	197	0	8	22.	464	12	2	36.	1645	2	6
9.	540	2	2	23.	118	10	9	37.	286	5	$11\frac{1}{4}$
10.	259	14	0	24.	1364	17	3	38.	251	12	$11\frac{1}{2}$
11.	71	8	$4\frac{3}{4}$	25.	148	4	$8\frac{1}{4}$	39.	2199	12	8
12.	82	16	1	26.	184	1	11	40.	289	8	3
13.	270	5	2	27.	435	19	10				
14.	362	2	$4\frac{1}{2}$	28.	87	8	$11\frac{1}{2}$				

Exercises 5.

	£	s.	d.
1.	13	0	$3\frac{3}{4}$
2.	4	12	$2\frac{3}{4}$
3.	8	8	5
4.	4	0	8
5.	4	19	4′
6.	8	11	$6\frac{1}{2}$
7.	8	6	3
8.	9	15	$11\frac{1}{4}$
9.	6	9	6
10.	1	15	$10\frac{3}{4}$
11.	7	15	$7\frac{1}{4}$
12.	0	19	$6\frac{1}{2}$
13.	4	15	0
14.	1	3	$5\frac{1}{2}$
15.	4	7	$0\frac{1}{2}$
16.	4	18	$8\frac{1}{4}$
17.	2	12	$8\frac{1}{4}$
18.	2	1	11
19.	0	14	$7\frac{1}{2}$
20.	3	6	3
21.	3	2	0
22.	5	7	$2\frac{1}{2}$
23.	1	11	$1\frac{3}{4}$
24.	0	18	$0\frac{1}{2}$

Exercises 6.

1. 15390136
2. 34069293
3. 343969014
4. 661292541
5. 107958378
6. 73905391
7. 20581596
8. 23743728
9. 1639830444
10. 4771801368
11. 2855587428
12. 856162450
13. 7256584665
14. 972118005
15. 18483904320
16. 50921667140
17. 221361822
18. 2039867550
19. 6599443356
20. 497064491
21. 4452724500
22. 118291774944
23. 1759792121472
24. 723129552
25. 16944932280
26. 13086255000

Exercises 7.

1. 75·12 lb.
2. 12·096 gall.
3. 665·664 c. ft.
4. 27·373 bu.
5. ·34578$\frac{2}{3}$; 86372
6. 70·9808; 1247038$\frac{3}{11}$
7. 6*s.* 7·123*d.*
8. 277·274 gall.
9. 14·15016 times; 1·886688 oz.
10. 38*s.* 7·249*d.*
11. ·0013163 –
12. 4·50776
13. 19*s.* 2$\frac{1}{2}$*d.* –
14. 36·0612 gall.
15. $65432\cdot\dot{1}\dot{0}$
16. 304·4137 –
17. 18·0327
18. 7*s.* 9·37*d.*—
19. 832630
20. 27083; 74·2
21. 9657·48 gall.
22. $\cdot 1345\dot{7}\dot{0}$

Exercises 8.

	£	s.	d.
1.	1	6	1·13
2.	5	14	7·83
3.	2	3	10·56
4.	2	5	1·17
5.	7	10	2·955
6.	30	9	2·047
7.	48	10	6·3
8.	1	4	1·9

Exercises 9.

	£	s.	d.
1.	209	12	6·24
2.	750	13	3·96
3.	118	16	0·56
4.	353	10	10·03
5.	1010	10	10
6.	57	3	3·97

MISCELLANEOUS QUESTIONS.

1\. 57
2\. £529 12*s.* 6*d.*
3\. 7 11 $5\frac{1}{2}$ +
4\. 17925 ;
19014·$05\frac{1}{3}$
5\. 36883098 ;
135432
6\. £799 5*s.* ;
£1010 11*s.* $10\frac{1}{2}$*d.*
7\. ·07812756 ;
570875
8\. 468
9\. £243 16*s.* $10\frac{1}{2}$*d.* ;
118 19 $4\frac{3}{4}$*d.* –
10\. 3509570·76 ;
43350
11\. 12*s.* 4·37*d.*
12\. £181 14*s.* $9\frac{1}{2}$*d.*
13\. 73
14\. 24747525 ;
50499450
15\. £130 15*s.* $7\frac{1}{2}$*d.* ;
146 11 $6\frac{3}{4}$*d.*
16\. 258781·152
17\. £81 11*s.* 9*d.*
18\. 32·70616 –
19\. 7587603 c. in. +
20\. £4 1*s.* $3\frac{1}{4}$*d.* ;
4 4 $5\frac{1}{2}$ +
21\. 67491411792 ;
103·$89\frac{1}{3}$
22\. ·869077
23\. 233
24\. 2974
25\. £57 7*s.* $3\frac{3}{4}$*d.*
26\. 389·69 +
27\. £9 5*s.* $2\frac{3}{4}$*d.*
28\. £310 13*s.* $6\frac{3}{4}$*d.* ;
962 7 8
29\. 34293552500
30\. 18229647861 ;
$2981\frac{1}{4}$
31\. £17 2*s.* $4\frac{1}{2}$*d.*
32\. 141
33\. £11 4*s.*
34\. £45 10*s.* 11*d.*
35\. 205·368 cubic feet
36\. £26 15*s.* 3*d.* ;
44 2 $2\frac{1}{4}$
37\. 6 ft. 7·146 in.
38\. £3 6*s.* 4*d.*
39\. £464 1*s.* $3\frac{3}{4}$*d.* +
40\. £194 5*s.* 0*d.* ;
104 8 $7\frac{1}{2}$
41\. 119
42\. £8 10*s.* $10\frac{1}{4}$*d.* ;
7 1 $8\frac{1}{2}$
43\. £1503 9*s.* $2\frac{1}{4}$*d.*
44\. 96·9068 oz.
45\. ·11372
46\. $2075\cdot\dot{7}\dot{0}$
47\. £8 15*s.* $10\frac{1}{4}$*d.*
48\. £11 11*s.* 11*d.* ;
5 17 $3\frac{3}{4}$
49\. 321·35 gall.
50\. 25·148 tons =
51\. £189 12*s.* $10\frac{1}{2}$*d.*
52\. 10·3746 ac. –
53\. 13
54\. $4932\frac{8}{81}$
55\. 223·36 gall. –
56\. $20615958\frac{7}{11}$
57\. £0 8*s.* $5\frac{1}{2}$*d.*
0 4 $5\frac{1}{4}$
58\. £107 2*s.* 10·84*d.*
59\. £10 1*s.* $5\frac{1}{2}$*d.*
60\. 4·292 ac.
61\. 37·567
62\. £48 3*s.* $4\frac{1}{4}$*d.*
63\. £73 8*s.* $4\frac{1}{2}$*d.*
64\. £536 7*s.* $3\frac{1}{4}$*d.*
12 14 10
65\. £106 7*s.* $11\frac{1}{2}$*d.*
66\. £3 0*s.* $5\frac{1}{4}$*d.* ;
0 5 $5\frac{3}{4}$
67\. 9·07 ft.
68\. £26 8*s.* 9*d.*
69\. £5400
70\. $16219791\cdot\dot{9}\dot{1}$
71\. £1 6*s.* $2\frac{1}{4}$*d.* ;
5 6 $1\frac{1}{2}$
72\. 59275 gallons nearly
73\. $23\frac{1}{13}$ p.c. disc.
74\. 2·82137
75\. $16\cdot58\dot{5}\dot{0}$
76\. 3·6 in. ; 3·7482 sq. in.
77\. 1068·42 yds.
78\. $3\frac{3}{8}$ ft.
79\. 2 hrs. $13\frac{9}{11}$ min.
80\. 36*s.*
81\. 841·45 c. in.
82\. 52 inches
83\. £30 8*s.* 8*d.*
84\. £4833 10*s.*
85\. 20*s.*
86\. 7·6418 p.c.
87\. 70.
88\. $\frac{202}{3310}$.
89\. 360 ; 20.
90\. $7\frac{1}{2}$ and $8\frac{3}{4}$
91\. 2 months.
92\. 7·23 ft.
93\. 9 gall.
94\. 25 in. ; 43·3 in.
95\. $42\cdot54\dot{1}\dot{2}$.
96\. 3548·5.
97\. $14\frac{2}{7}$
98\. 4 ft. 7·015 in.
99\. A. 152·46 hrs.
B. $155\frac{4}{7}$ hrs.
100\. £580

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